MTHSC 206 Section 17.1 – Vector Fields

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Definition

1. Let $D \subseteq \mathbb{R}^2$. A vector field on D is a function F that assigns to each point $(x, y) \in D$, a vector $F(x, y) \in \mathbb{R}^2$.
2. Let $E \subseteq \mathbb{R}^3$. A vector field on E is a function F that assigns to each point $(x, y, z) \in E$, a vector $F(x, y, z) \in \mathbb{R}^3$.

Note

We typically write 2 dimensional vector fields as $F(x, y) = P(x, y)i + Q(x, y)j$ and 3 dimensional vector fields as $F(x, y, z) = P(x, y, z)i + Q(x, y, z)j + R(x, y, z)k$, where P, Q and R are real valued functions.

Example

Plot $F(x, y) = (-y, x) = -yi + xj$. Note that F assigns to each vector an orthogonal vector.

Definition

1. Suppose that $f(x, y)$ is a differentiable function on $\mathbb{R}^2$. Then ∇f is a vector field on $\mathbb{R}^2$ called a gradient field.
2. Suppose that $g(x, y, z)$ is a differentiable function on $\mathbb{R}^3$. Then ∇g is a vector field on $\mathbb{R}^3$ called a gradient field.

Example

Consider the function $f(x, y) = x^2 + y^2$. Plot the gradient field for f and several contours of f.

Definition

A vector field F is called a conservative field if it is the gradient field for some function f.
In the case that $F = \nabla f$, we say that f is a potential function for F.