

MTHSC 206 SECTION 17.9 – THE DIVERGENCE THEOREM

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DEFINITION

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THE DIVERGENCE THEOREM

Let E be a simple solid region and let $S = \partial E$ be the boundary of E with positive orientation. Let $F = [P, Q, R]$ be a vector field whose components have continuous partials on an open region containing E . Then

$$\int \int_S F \cdot d\mathbb{S} = \int \int \int_E \operatorname{div}(F) \, dV$$

EXAMPLE

Find the flux of the vector field $F(x, y, z) = [z, y, x]$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

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Evaluate $\int \int_S F \cdot d\mathbf{S}$, where $F(x, y, z) = [xy, y^2 + e^{xz^2}, \sin(xy)]$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$, the xy -plane, the xz -plane and the plane $y + z = 2$.

NOTE

We can apply the divergence theorem to other regions as well.

- 1 If $E = E_1 \cup E_2 \cup \cdots \cup E_k$ is the union of a finite number of simple solids, then the divergence theorem will still hold for E .

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- 1 If $E = E_1 \cup E_2 \cup \cdots \cup E_k$ is the union of a finite number of simple solids, then the divergence theorem will still hold for E .
- 2 Suppose that S_1 is a closed surface lying inside the closed surface S_2 and the E is the region between the two. Then,

$$\int \int \int_E \operatorname{div}(F) \, dV = \int \int_{S_2} F \cdot d\mathbf{S} - \int \int_{S_1} F \cdot d\mathbf{S}$$