MTHSC 3190 Section 1.4

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Proposition

The sum of two even integers is even.

Proof.

1. We will prove: If x and y are even integers then $(x + y)$ is also even.
2. Let x and y be even integers.
3. Since x is even, by the definition of even, $2|x$.
4. Likewise, since y is even, by the definition of even, $2|y$.
5. Since $2|x$, we know by the definition of even that there is an integer a such that $x = 2a$.
6. Similarly, since $2|y$, there is an integer b such that $y = 2b$.
7. Observe that $x + y = 2a + 2b = 2(a + b)$
8. Since a and b are integers, $a + b$ is an integer
9. Therefore, there is an integer c (namely $c = a + b$) such that $x + y = 2c$.
10. Therefore, by the definition of divisibility, $2|(x + y)$.

Proof Template for If-then statements

To prove If A then B:

1. If necessary, rewrite the fact to be proved in if-then form with appropriate notation.
2. For the first sentence of the proof, rewrite the hypothesis A, introducing appropriate notation. Be careful to use different variable names for different objects.
3. The last sentence of the proof should be a restatement of the conclusion B.
4. Working from the top, unravel the definitions of words used in the previous statements.
5. Working from the bottom, unravel the definitions of words appearing below.
6. **Key Step** Forge a link between the two ends.

Proposition

Let x be an integer. If $x > 1$ then $(x^3 + 1)$ is composite.

Proof.

Exercise. $\square$

Proposition

Let a, b and c be integers. If $a|b$ and $b|c$, then $a|c$.

Proof.

Exercise. $\square$

Recall

We recall that the statement $\underline{A \text{ if and only if } B}$ is equivalent to the two statements

1. If A then B.
2. If B then A.

So, the following proof template should be no surprise.

Proof Template for If-and-only-if statements

To prove $\underline{A \text{ if and only if } B}$: Use the previous proof template to show

1. If A then B and,
2. If B then A.

Proposition

Let x be an integer. The integer x is even if and only if the integer $(x+1)$ is odd.

Proof.

Exercise $\square$

Note

It is sometimes convenient to recall previous results in our proof. Consider the following example.

Proposition

Let a, b, c and d be integers. If $a|b$, $b|c$ and $c|d$, then $a|d$.

Proof.

Exercise. □

Example

Let's prove the following.

Fact

If x and y are integers then $(x+y)^2 \geq 4xy$.

NON proof

$(x+y)^2 \geq 4xy$
$x^2 + 2xy + y^2 \geq 4xy$
$x^2 - 2xy + y^2 \geq 0$
$(x-y)^2 \geq 0 \rightarrow \text{TRUE}.$

Make this scratch work into a proof if possible...