MTHSC 3190 Section 1.5 Counterexample

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Note

In general, to disprove a statement one needs to prove the logical negation of the statement. Since this is particularly simple in the case of If-Then statements, we give a technique for refuting false If-Then statements here.

Example

Proposition (FALSE!!!)

If x is a perfect square, then x is positive.

Disproof

Note that this false statement asserts that all squares are positive. Thus, to show that this is a false mathematical statement, we simply need to show one instance (a counter-example) of a non-positive square.
How about $0 = 0 \times 0$ which is not positive.

Proof Template for refuting false If-Then statements

To refute a false statement of the form If A then B:

Find an instance where A is true and B is false.

Example

Disprove the false statement

False Statement

Let a and b be integers. If $a|b$ and $b|a$, then $a = b$.