MTHSC 3190 Section 1.6 Boolean Algebra

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Boolean Operators

The Boolean operators $\wedge$, $\vee$ and $\neg$ referred to as And, Or and Not are defined by the following truth tables.

A	B	$A \wedge B$	$A \vee B$
F	F	F	F
F	T	F	T
T	F	F	T
T	T	T	T

A	$\neg A$
F	T
T	F

Example

Calculate the value of

$$((\neg \text{False}) \vee (\neg \text{True})) \wedge \text{True}.$$

Definition

Two boolean expressions are logically equivalent provided the have the same truth-values for all possible instances of their variables.

Proposition

$\neg(x \wedge y) = (\neg x) \vee (\neg y)$.

Proof.

Exercise $\square$

Proof Template for Boolean Expressions

To show the equivalence of two boolean expressions:

1. First, we write a sentence of explanation such as "In order to compare the above boolean expressions in all possible instances of their variable set, we construct the following truth table.
2. Construct a table showing the values of the two statements for all possible instances of their variable sets. (If the statements have n variables, you will need 2^n rows).
3. Check that the two expressions always agree or note that they fail to agree.
4. Write a sentence stating that the expressions always agree or note that you have disproved the equivalence and point out the counterexample.

Theorem (6.2)

1. $x \wedge y = y \wedge x$; $x \vee y = y \vee x$.
2. $(x \wedge y) \wedge z = x \wedge (y \wedge z)$; $(x \vee y) \vee z = x \vee (y \vee z)$.
3. $x \wedge (True) = x$; $x \vee False = x$.
4. $\neg(\neg x) = x$.
5. $x \wedge x = x$; $x \vee x = x$.
6. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$;
   $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.
7. $x \wedge (\neg x) = False$;
   $x \vee (\neg x) = True$.
8. $\neg(x \wedge y) = \neg x \vee \neg y$;
   $\neg(x \vee y) = \neg x \wedge \neg y$.

Proof.

Exercise $\square$

Implication Operators

The operators $\rightarrow$ and $\leftrightarrow$ are defined by

A	B	$A \rightarrow B$	$A \leftrightarrow B$
F	F	T	T
F	T	T	F
T	F	F	F
T	T	T	T

Proposition

1. $x \to y = \neg x \vee y$.
2. $x \to y = \neg y \to \neg x$.

Proof.

Exercise $\square$