MTHSC 3190 Section 2.7 Collections

Kevin James

Definition

A list is an ordered sequence of objects.

Example

- $(1, 2, 3)$
- $(1, 1, 2, 3, 4, 3, 1)$
- $(1, 2, x, s, \mathcal{S}, \mathbb{Z})$

Definition

The length of the list is the number of objects in the list.

Example

- $(1, 2, 3)$ has length 3.
- $(1, 2, 2, 1)$ has length 4.
- $()$ denotes the empty list and has length 0.

Equality of Lists

Two lists are considered to be equal provided that they have the same length and that the elements in corresponding positions are the same.

Note

The above definition requires that we are able to discern equality of the objects allowed to appear on our lists.

Example

- $(1, 2, 3) = (1, 2, 3)$,
- $(1, 2, 3) \neq (1, 2, 3, 4)$,
- $(1, 2, 3) \neq (1, 3, 2)$.

Definition

Lists of length 2 are referred to as ordered pairs.

Counting Lists

How many ordered pairs whose elements are 1,2 or 3 are there?

Fact

The number of ordered pairs whose elements are chosen from $1, 2, 3, \ldots, n$ *is* n^2.

Theorem (Multiplication Principle -1^{st} version)

The number of ordered pairs that can be constructed when the first element is chosen from among n items and the second element is chosen from m items is nm.

Example

List all ordered pairs with the first element chosen from 1,2 or 3 and the second element chose from A or B.

Example

List all ordered pairs with the first element chosen form 1,2 or 3 and the second element chosen from

$$\begin{cases} A \text{ or } B & \text{if the first element is 1,} \\ C \text{ or } D & \text{if the first element is 2,} \\ E \text{ or } F & \text{if the first element is 3.} \end{cases}$$

Proof of Multiplication Principle

Example

List all ordered pairs whose elements come form 1,2,3 or 4 and repeats are not allowed.

Corollary

The number of ordered pairs without repeats whose elements are from $\{1, 2, \ldots, n\}$ *is* $n(n-1)$.

Example

Find all lists of length 3 whose elements are 1, 2 or 3.

Proposition

The number of 3 element lists where the elements are $1, 2, \ldots, (n-1)$ *or* n *is* n^3.

Example

Find all lists of length 3 without repeats whose elements are 1, 2, 3 or 4.

Proposition

The number of 3 element lists without repeats where the elements are $1, 2, \ldots, (n-1)$ *or* n *(for* $n \geq 3$*) is* $n(n-1)(n-2)$.

General Multiplication Principle

Consider lists of length k where there are a_j choices for the j^{th} element where $1 \leq j \leq k$ and $a_j \in \mathbb{N}$. The number of such lists is

$$\prod_{j=1}^{k} a_j = a_1 \cdot a_2 \cdot \cdots \cdot a_k.$$

Proof.

We will induct on n. $\square$

Definition

For $n, k \in \mathbb{N}$, we define

$$(n)_k = \prod_{j=0}^{k-1}(n-j) = n \cdot (n-1) \cdot \dots \cdot (n-k+1).$$

Corollary

The number of length k lists whose elements are $1, 2, \dots, (n-1)$ or n is

$$\begin{cases} n^k & \text{if repeats are allowed, and} \\ (n)_k & \text{if repeats are not allowed.} \end{cases}$$