MTHSC 3190 Section 2.7 Collections

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Definition

A list is an ordered sequence of objects.

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Example

- $(1, 2, 3)$
- $(1, 1, 2, 3, 4, 3, 1)$
- $(1, 2, x, s, \mathcal{S}, \mathbb{Z})$

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Example

- $(1, 2, 3)$ has length 3.
- $(1, 2, 2, 1)$ has length 4.
- $()$ denotes the empty list and has length 0.

Equality of Lists

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Example

- $(1,2,3) = (1,2,3)$,
- $(1,2,3) \neq (1,2,3,4)$,
- $(1,2,3) \neq (1,3,2)$.

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The number of ordered pairs whose elements are chosen from $1, 2, 3, \ldots, n$ *is* n^2.

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The number of ordered pairs whose elements are chosen from $1, 2, 3, \ldots, n$ *is* n^2.

Theorem (Multiplication Principle -1^{st} version)

The number of ordered pairs that can be constructed when the first element is chosen from among n items and the second element is chosen from m items is nm.

Example

List all ordered pairs with the first element chosen from 1,2 or 3 and the second element chose from A or B.

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$$\begin{cases} A \text{ or } B & \text{if the first element is 1,} \\ C \text{ or } D & \text{if the first element is 2,} \\ E \text{ or } F & \text{if the first element is 3.} \end{cases}$$

Proof of Multiplication Principle

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Example

List all ordered pairs whose elements come form 1,2,3 or 4 and repeats are not allowed.

Corollary

The number of ordered pairs without repeats whose elements are from $\{1, 2, \ldots, n\}$ *is* $n(n-1)$.

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Proposition

The number of 3 element lists where the elements are $1, 2, \ldots, (n-1)$ *or* n *is* n^3.

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Proposition

The number of 3 element lists without repeats where the elements are $1, 2, \ldots, (n-1)$ *or* n *(for* $n \geq 3$*) is* $n(n-1)(n-2)$*.*

General Multiplication Principle

Consider lists of length k where there are a_j choices for the j^{th} element where $1 \leq j \leq k$ and $a_j \in \mathbb{N}$. The number of such lists is

$$\prod_{j=1}^{k} a_j = a_1 \cdot a_2 \cdot \cdots \cdot a_k.$$

Proof.

We will induct on n. □

Definition

For $n, k \in \mathbb{N}$, we define

$$(n)_k = \prod_{j=0}^{k-1}(n-j) = n \cdot (n-1) \cdot \cdots \cdot (n-k+1).$$

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Corollary

The number of length k lists whose elements are $1, 2, \ldots, (n-1)$ or n is

$$\begin{cases} n^k & \textit{if repeats are allowed, and} \\ (n)_k & \textit{if repeats are not allowed.} \end{cases}$$