MTHSC 3190 Section 3.13

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Definition

A relation is a set of ordered pairs.

Example

$$R = \{(0,0),(0,1),(1,2),2,5)\}$$

Notation

If $(x,y) \in R$, we say "x is related to y by R" and we may write xRy. If $x,y) \notin R$ we write $x \not R y$.

Example

The $<$ relation can be thought of as the set

$$< \; = \{\ldots, (-2,-1), (-2,0), \ldots, (1,2), \ldots (3,100), \ldots\}$$

Definition

Let R be a relation and let A and B be sets.

1. We say that "R is a relation on A" provided that $R \subseteq A \times A$.
2. We say that "R is a relation from A to B" provided that $R \subseteq A \times B$.

Example

Let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ and let
$R = \{(1, 1), (2, 2), (3, 3)\}$,
$S = \{(1, 2), (2, 3)\}$,
$T = \{(2, 4), (3, 5)\}$,
$U = \{(4, 1), (3, 2), (5, 3)\}$,
$V = \{(17, 32), (0, 5), (1, 3), (2, 5)\}$,
Then,

R is a relation ________________
S is a relation ________________
T is a relation ________________
U is a relation ________________
V is a relation ________________

Definition

Let R be a relation. The inverse of R denoted by R^{-1} is the relation formed by reversing the order of all pairs in R.
(-i.e. $R^{-1} = \{(y, x) \ : \ (x, y) \in R\}$.)

Example

Let $R = \{(1,1),(1,2),(1,3),(2,4),(4,2)\}$. What is R^{-1}?

Note

If R is a relation from A to B, then R^{-1} is a relation from B to A.

Proposition

Let R be a relation. Then $\left(R^{-1}\right)^{-1} = R$.

Proof

Definition

Let R be a relation defined on a set A.

1. If $\forall x \in A, xRx$, then R is reflexive.
2. If $\forall x \in A, x \not R x$, then R is irreflexive.
3. If $\forall (x, y) \in R, (y, x) \in R$, then R is symmetric.
4. If $\forall x, y \in A, (xRy \wedge yRx) \Rightarrow x = y$, then R is antisymmetric.
5. If $\forall x, y \in A, (xRy \wedge yRz) \Rightarrow xRz$, then R is transitive

Example

Let $A = \{1, 2, 3, 4\}$. For each of the following relations on A, indicate which properties the relation has and does not have.

1. $R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$
2. $S = \{(1, 1), (1, 2), (2, 1), (3.2), (2, 3)\}$
3. $T = \{(1, 2), (1, 3), (2, 3)\}$
4. $U = \{(1, 1), (2, 3), (3, 1), (1, 2)\}$

Example

Some other relations to consider.

1. $<$ on $\mathbb{Z}$.
2. $\leq$ on $\mathbb{Z}$.
3. $|$ on $\mathbb{N}$.
4. $|$ on $\mathbb{Z}$.