MTHSC 3190 Section 3.16

Kevin James

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Now, let's carefully compute $\binom{5}{2} = \binom{5}{3}$.

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Example

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Binomial Theorem

Let $n \in \mathbb{N}$. Then

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

Proof.

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Exercise

Arrange the binomial coefficients into a triangle. This is known as Pascal's tirangle. Write out the first few rows. Do you notice a relationship between consecutive rows?

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Theorem

Let $n, k \in \mathbb{N}$*. Then* $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Combinatorial Proof.

□

A formula for $\binom{n}{k}$

Theorem

Suppose that $n, k \in \mathbb{N}$. Then

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Proof.

$\square$