Thm 18.1 (Inclusion/Exclusion):

Let $A_1, \dots, A_n$ be finite sets. Then

$$|A_1 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n|$$
$$- |A_1 \cap A_2| - |A_1 \cap A_3| \dots - |A_{n-1} \cap A_n|$$
$$+ |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \dots + |A_{n-2} \cap A_{n-1} \cap A_n|$$
$$- \dots$$
$$+ \dots$$
$$\vdots$$
$$\pm |A_1 \cap \dots \cap A_n|$$

- eg.: $|A \cup B \cup C \cup D| =$

Proof: Suppose $A_1, \dots, A_n$ are finite sets.

Let $A_1 \cup \dots \cup A_n = \{x_1, x_2, \dots, x_m\}$

Construct a chart as follows

Element	A_1	A_2	...	A_n	$A_1 \cap A_2$	...	$A_{n-1} \cap A_n$	$A_1 \cap A_2 \cap A_3$	...	$A_1 \cap A_2 \cap \dots \cap A_n$
1										
2										
⋮										
m										

In the ij^{th} entry we place:

$$\begin{cases} 0 & \text{if } x_i \notin \text{set at top of } j^{th} \text{ column.} \\ (-1)^k & \text{if } x_i \in \text{set at top of } j^{th} \text{ columns and if this set has } k \text{ intersections.} \end{cases}$$

*Note: The sum of entries in the i^{th} row is the # of times that x_i is counted in the RHS of our formula.

So, we need to show that the sum of the entries in each row is 1.

If $x_i \in k$ of the sets $A_1, \ldots, A_n$

Then in row i we have:

$\#\ +'s =$

$\#\ -'s =$

$\therefore$ sum of entries in row i is:

List Counting Problem:

How many of the n^k k-element lists with entries chosen from $\{1, \ldots, n\}$ and allowing repeats have each number in $\{1, 2, \ldots, n\}$ appearing at least once.

Answer: Let

$$U = \{\text{all k-element lists from } \{1,\ldots,n\} \text{ allowing repeats}\}$$

$$G = \text{good lists} = \{\text{k-element lists which use each element of } \{1,\ldots,n\} \text{ at least once}\}$$

$$B = \text{Bad lists} = \{\text{lists which miss at least one number}\}$$

Note: $\# G = n^k - \# B$.

We will use Inc/Exc, to compute $\# B$.

Let $B_i = \{\text{k-element lists which are missing } i\}$

Then $B = B_1 \cup B_2 \cup \cdots \cup B_n$

Observations:

1. $|B_i| =$

There are $n = \binom{n}{\_}$ of these.

2. For $i \neq j$,

$|B_i \cap B_j| =$

There are ______ of these

3. If $\{i_1, \ldots, i_\ell\} \subseteq \{1, 2, \ldots, n\}$ then

$|B_{i_1} \cap B_{i_2} \cap \cdots \cap B_{i_\ell}| =$

There are ______ such intersections.

So, $|B| = |B_1 \cup \cdots \cup B_n| =$