MTHSC 3190 Section 4.19

Kevin James

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Note

The challenge with proving the statement "If m^2 is even, then m is even" directly is that writing $m^2 = 2k$ isn't enough. For example, 6 is even, but it is not a square. A proof by the contrapositive makes the statement much easier to prove.

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Let x be an integer. If $x^2 + 2x < 0$, then $x < 0$.

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For the sake of contradiction, assume that $\neg S$ is true.

$$\vdots$$

Deduce a statement which is obviously false.
Then you have proved that $(\neg S \Rightarrow \text{FALSE})$ and thus $\neg S$ is false which means that S is true. □

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Thus, $2x = x + x = 2k + 2m + 1$.

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Thus our assumption must have been false so we have proved the

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Proposition

Suppose that $a, b \in \mathbb{Z}$. If $a \neq 0$, then there is at most one integer x such that $ax + b = 0$.