MTHSC 3190 Section 4.21

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Theorem (Principle of Mathematical Induction)

Let $A \subseteq \mathbb{N}$*. If*

1. $0 \in A$.
2. $\forall k \in \mathbb{N}, k \in A \Rightarrow k+1 \in A$.

then

$$A = \mathbb{N}.$$

Proof.

$\square$

Proof by Induction

To prove that every natural number has some property:

1. Let A be the set of natural numbers having the desired property.
2. *Basis Step* Prove that $0 \in A$.
3. Prove the statement $(k \in A \Rightarrow (k+1) \in A)$.
   Note: *The hypothesis that $k \in A$ is called the induction hypothesis. The proof of the above statement is called the induction step.*
4. Use the Principle of Mathematical Induction to conclude that $A = \mathbb{N}$.

Proposition

Let $n \in \mathbb{N}$. Then,

$$\sum_{k=0}^{n} k^2 = \frac{(2n+1)(n+1)n}{6}.$$

Proof.

We will proceed by induction on n.

$\square$

Proposition

Let $n \in \mathbb{N}$. Then,

$$\sum_{k=0}^{n-1} 2^k = 2^n - 1.$$

Proof.

$\square$

Proposition

Let $n \geq 1$. Then,

$$1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1.$$

Proof.

$\square$

Proposition

Let $n \in \mathbb{N}$. Then, $3 | (4^n - 1)$.

Proof.

$\square$

Principle of Strong Mathematical Induction

Let $A \subseteq \mathbb{Z}$. Suppose that we know the following.

1. $M \in A$.
2. $\{M, M+1, \ldots, k\} \subseteq A \Rightarrow (k+1) \in A$.

Then $A = \{x \in \mathbb{Z} \ : \ x \geq M\}$.

Proof by Strong Induction

To prove all integers $x \geq M$ have some property P:

1. Let A be the set of integers having the desired property.
2. *Basis Step* Prove that $M \in A$.
3. Prove the statement
   $(\{M, M+1, \ldots, k\} \subseteq A \Rightarrow (k+1) \in A)$.
   Note: *The hypothesis that $k \in A$ is called the* *induction hypothesis. The proof of the above statement is called the induction step.*
4. Use the Principle of Strong Mathematical Induction to conclude that $A = \{x \in \mathbb{Z} \ : \ x \geq M\}$.

Proposition

If a polygon with 4 or more sides is triangulated then at least two of the triangles formed are exterior.

Proof.

□

Proposition

Let $n \in \mathbb{N}$. Then,

$$\sum_{j=0}^{n} \binom{n-j}{j} = F_n.$$

Proof.

□