MAT 3190
Exercises
Due: 3:00 p.m. Friday, November 15, 2013

Name:____

You may use your notes and your books. You may also work in groups. However, please write up your own version of the work (-i.e. no mindless copying). Please show all of your work. An answer without justification will receive little credit.

(1) Suppose that A and B are sets and that $f : A \to B$ is a function. Show that the inverse relation f^{-1} is a function if and only if f is injective (one to one). Show that when f^{-1} is a function it is injective. (**Hint:** For the 2nd part, it will be useful to remember that f is a well defined function.)

(2) Suppose that $f : A \to B$ is a function and that f^{-1} is also a function. Show that $\mathrm{dom}(f) = \mathrm{im}(f^{-1})$ and $\mathrm{im}(f) = \mathrm{dom}(f^{-1})$.

(3) Suppose that $f : A \to B$. Show that $f^{-1} : B \to A$ if and only if f is bijective. (**Hint:** Think carefully about what the statement $f^{-1} : B \to A$ means.)

(4) Define $\Gamma : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ (where $\mathbb{R}_{>0}$ denotes the positive real numbers) by

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} \mathrm{dx}.$$

Show by induction on n that $\Gamma(n) = (n-1)!$ for all integers $n \geq 1$. (**Hint:** For the induction step, write down the formula for $\Gamma(n)$ and use integration by parts.)