MTHSC 3190 Section 5.24

Kevin James

Proposition

Let $n \in \mathbb{N}$. Then there exists integers $a \neq b$ such that $10 | (n^a - n^b)$.

Proof.

$\square$

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Given 5 distinct integer lattice points in $\mathbb{R}^2$, at least on of the line segments determined by these points has an integral midpoint.

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Proposition (Cantor)

Let A be a set. If $f : A \to 2^A$. then f is **NOT** *onto.*

Note

If A is finite then $|A| < |2^A| = 2^{|A|}$ and the result follows trivially from the pigeon hole principle.

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Let A be a set. We will find $B \in 2^A$ (-i.e. $B \subseteq A$) such that $B \notin \text{im}(f)$.

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Claim: $B \notin \text{im}(f)$.
Proof: □