MTHSC 3190 Section Limits and Functions

Kevin James

Definition (The Limit)

Suppose that $f : \mathbb{R} \to \mathbb{R}$ and that $a, L \in \mathbb{R}$. We write

$$\lim_{x \to a} f(x) = L.$$

if $\forall \epsilon > 0, \exists \delta > 0, \forall |x - a| < \delta, |f(x) - L| < \epsilon$.

Example

1. Show that $\lim_{x \to 3}(5x + 2) = 17$.
2. Show that $\lim_{x \to 2}(x^2 + 2) = 6$.

Definition

Suppose that $f : \mathbb{R} \to \mathbb{R}$ and that $a \in \mathbb{R}$.

1. We say that f is continuous at a if

$$\lim_{x \to a} f(x) = f(a).$$

2. We say that f is continuous everywhere in $\mathbb{R}$ if
   $\forall a \in \mathbb{R}, f$ is continuous at a.

Example

1. Show that $f(x) = x + 3$ is continuous at 4.
2. Show that $f(x) = x^2 + 2$ is continuous at 1.
3. Show that $f(x) = x + 1$ is continuous everywhere.

EXERCISE

1. Show that $f(x) = 5x + 3$ is continuous at 2.
2. Show that $f(x) = 5x + 3$ is continuous everywhere in $\mathbb{R}$.

Suppose that $* : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a binary operator on $\mathbb{R}$.

1. If, $(a, b) \in \mathbb{R}^2$ and $L \in \mathbb{R}$, we write

$$\lim_{(x,y) \to (a,b)} x * y = L$$

if $\forall \epsilon > 0, \exists \delta > 0, \forall (x, y)$ satisfying $\sqrt{(x-a)^2 + (y-b)^2} < \delta, |x * y - L| < \epsilon$.

2. If $(a, b) \in \mathbb{R}^2$, we say that $*$ is continuous at (a, b) if $\lim_{(x,y) \to (a,b)} x * y = a * b$.

3. We say that $*$ is a continuous operation on $\mathbb{R}$ if $\forall (a, b) \in \mathbb{R}^2, *$ is continuous at (a, b).

EXERCISE

1. Show that addition is a continuous operation on $\mathbb{R}$.

2. Show that multiplication is a containuous operation on $\mathbb{R}$.