MTHSC 3190 Section Limits and Functions

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Definition (The Limit)

Suppose that $f : \mathbb{R} \to \mathbb{R}$ and that $a, L \in \mathbb{R}$. We write

$$\lim_{x \to a} f(x) = L.$$

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Example

1. Show that $\lim_{x \to 3}(5x + 2) = 17$.
2. Show that $\lim_{x \to 2}(x^2 + 2) = 6$.

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2. Show that $f(x) = x^2 + 2$ is continuous at 1.
3. Show that $f(x) = x + 1$ is continuous everywhere.

Exercise

1. Show that $f(x) = 5x + 3$ is continuous at 2.
2. Show that $f(x) = 5x + 3$ is continuous everywhere in $\mathbb{R}$.

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1. If, $(a, b) \in \mathbb{R}^2$ and $L \in \mathbb{R}$, we write

$$\lim_{(x,y)\to(a,b)} x * y = L$$

if $\forall \epsilon > 0, \exists \delta > 0, \forall (x, y)$ satisfying $\sqrt{(x-a)^2 + (y-b)^2} < \delta, |x * y - L| < \epsilon$.

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1. Show that addition is a continuous operation on $\mathbb{R}$.
2. Show that multiplication is a containuous operation on $\mathbb{R}$.