

HELPFUL HINTS FOR LIMITS AND CONTINUITY

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The following observations will be helpful in completing your homework assignment on the Limits and Continuity section.

(1) For the problems concerning continuity of operators it will be helpful to note that

$$\begin{aligned} & \sqrt{(x-a)^2 + (y-b)^2} < \delta \\ \Rightarrow & (x-a)^2 + (y-b)^2 < \delta^2 \\ \Rightarrow & (x-a)^2 < \delta^2 \quad \text{and} \quad (y-b)^2 < \delta^2 \quad \text{because both } (x-a)^2 \geq 0 \text{ and } (y-b)^2 \geq 0. \\ \Rightarrow & |x-a| < \delta \quad \text{and} \quad |y-b| < \delta. \end{aligned}$$

(2) Also, it may be helpful to use the triangle inequality which is

$$\forall A, B \in \mathbb{R}, \quad |A + B| \leq |A| + |B|.$$

This can of course be applied to any expressions A and B involving variables which represent real numbers.

(3) When proving continuity of multiplication, it may be helpful to note that

$$|xy - ab| = |(x-a)(y-b) + b(x-a) + a(y-b)| \leq |x-a||y-b| + |b||x-a| + |a||y-b|.$$

The inequality follows from the triangle inequality (above).