MTHSC 412 Section 1.1

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Note

We avoid repetition when listing the objects of a set since for example $\{1, 2\} = \{1, 2, 2\}$.

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Example

Let

$$A = \{1, 2, 5, 7\}$$

Then $2 \in A$ but $8 \notin A$.

We will frequently describe a set by giving the types of objects allowed in our set followed by some additional properties which must be satisfied for membership. The description of allowable objects is usually separated from the list of properties by a : or a $|$.

$$\{\text{allowable elements} \quad | \quad \text{required properties}\}$$

Fore example $B = \{x \in \mathbb{Z} \quad | \quad x \geq 0\}$ denotes the set of all integers x such that $x \geq 0$.

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2. Two sets A and B are said to be equal if they contain exactly the same elements (i.e. if $A \subseteq B$ and $B \subseteq A$).
3. If $A \subseteq B$ but $A \neq B$ then A is said to be a proper subset of B. In this case we will write $A \subset B$.

Example

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Then $\{a, b\} \subseteq A$ but $\{a, z\} \not\subseteq A$.
2. Note that $b \in A$ and $\{b\} \subseteq A$.

Definition (Intersection and Union)

Let A and B be sets. Then we define their *union* and *intersection* as follows:

- $A \cup B = \{x \quad | \quad x \in A \quad \text{or} \quad x \in B\}$.
- $A \cap B = \{x \quad | \quad x \in A \quad \text{and} \quad x \in B\}$.

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Example

Let

$$A = \{1, 3, 5, 6, 7\}; \qquad B = \{2, 4, 5, 6, 7\}.$$

Then,

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\} \quad \text{and} \quad A \cap B = \{5, 6, 7\}$$

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Facts

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2. *(Associativity)* $(A \cup B) \cup C = A \cup (B \cup C)$ *and* $(A \cap B) \cap C = A \cap (B \cap C)$

Definition (Empty Set, Disjoint, Pairwise Disjoint)

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Example

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1. Let $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$. Then A and B are disjoint.
2. Let $A_1 = \{1\}$, $A_2 = \{2\}$ and for $i \geq 1$, let $A_i = \{i\}$. Then the collection $A_1, A_2, \ldots$ is pairwise disjoint.

Definition (Power Set)

Let A be a set. The *power set* of A denoted $\mathcal{P}(A)$ is a set whose elements are the subsets of A. That is

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Note

For any set A, we have $\emptyset, A \in \mathcal{P}(A)$ or $\{\emptyset, A\} \subseteq \mathcal{P}(A)$.

Definition (Set Difference and Complements)

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2. In may cases all sets under consideration will be subsets of some *universal set* U. In this case, we define the *complement* A' of a set $A \subseteq U$ as

$$A' = \{x \in U \quad | \quad x \notin A\}.$$

Example

Let

$$\begin{aligned} U &= \mathbb{Z}, \\ E &= \{x \in \mathbb{Z} \mid x \text{ is even}\}, \\ O &= \{x \in \mathbb{Z} \mid x \text{ is odd}\} \quad \text{and} \\ P &= \{x \in \mathbb{Z} \mid x > 0\}. \end{aligned}$$

Then

$$\begin{aligned} P - E &= \{1, 3, 5, 7, 9, \ldots\} \\ E - P &= \{0, -2, -4, -6, \ldots\} \\ E' &= O, \quad \text{and} \\ O' &= E. \end{aligned}$$

Definition (Partitions)

A separation of a set into nonempty pairwise disjoint sets is a partition.

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Example

Suppose $A = \{1, 2, 3, 4, 5, 6\}$ and $X_1 = \{1, 4\}$, $X_2 = \{2, 3, 5\}$ and $X_3 = \{6\}$. Then $\mathcal{P} = \{X_1, X_2, X_3\}$ is a partition of A into 3 parts.

Properties of Union and Intersection

Facts

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
3. $(A \cap B)' = A' \cup B'$.
4. $(A \cup B)' = A' \cap B'$.