MTHSC 412 Section 1.1

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Definition

A *set* is a collection of objects with the property that it is possible to determine if a given object is in the set or not.

Example

1. $A = \{0, 1, 2, 3\}$.
2. $B = \{0, 1, 2, 3, \ldots\}$.

Note

We avoid repetition when listing the objects of a set since for example $\{1, 2\} = \{1, 2, 2\}$.

Notation

1. $\mathbb{Z}$ represents the set of integers.
2. $\mathbb{Q}$ represents the rational numbers.
3. $\mathbb{R}$ represents the real numbers.
4. $\mathbb{C}$ represents the complex numbers
5. $x \in S$ is read x is an element of S.
6. $x \notin S$ is read x is not an element of S.

Example

Let

$$A = \{1, 2, 5, 7\}$$

Then $2 \in A$ but $8 \notin A$.

We will frequently describe a set by giving the types of objects allowed in our set followed by some additional properties which must be satisfied for membership. The description of allowable objects is usually separated from the list of properties by a : or a $|$.

$$\{\text{allowable elements} \quad | \quad \text{required properties}\}$$

Fore example $B = \{x \in \mathbb{Z} \quad | \quad x \geq 0\}$ denotes the set of all integers x such that $x \geq 0$.

Definition

1. If A and B are sets we say that A is a subset of B and write $A \subseteq B$ if each element of A is also an element of B.
2. Two sets A and B are said to be equal if they contain exactly the same elements (i.e. if $A \subseteq B$ and $B \subseteq A$).
3. If $A \subseteq B$ but $A \neq B$ then A is said to be a proper subset of B. In this case we will write $A \subset B$.

Example

1. Let
$$A = \{a, b, c, d, e\}.$$
Then $\{a, b\} \subseteq A$ but $\{a, z\} \not\subseteq A$.
2. Note that $b \in A$ and $\{b\} \subseteq A$.

Definition (Intersection and Union)

Let A and B be sets. Then we define their *union* and *intersection* as follows:

- $A \cup B = \{x \quad | \quad x \in A \quad \text{or} \quad x \in B\}$.
- $A \cap B = \{x \quad | \quad x \in A \quad \text{and} \quad x \in B\}$.

Example

Let

$$A = \{1, 3, 5, 6, 7\}; \qquad B = \{2, 4, 5, 6, 7\}.$$

Then,

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\} \quad \text{and} \quad A \cap B = \{5, 6, 7\}$$

.

Facts

1. *(Commutativity)* $A \cup B = B \cup A$ *and* $A \cap B = B \cap A$.
2. *(Associativity)* $(A \cup B) \cup C = A \cup (B \cup C)$ *and* $(A \cap B) \cap C = A \cap (B \cap C)$

Definition (Empty Set, Disjoint, Pairwise Disjoint)

1. The empty set is the set with no elements and is denoted by $\emptyset$ or $\{\}$.
2. Two sets A and B are said to be disjoint if $A \cap B = \emptyset$.
3. A collection of sets $A_1, A_2, \ldots$ is said to be pairwise disjoint if any two sets in the collection are disjoint.

Example

1. Let $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$. Then A and B are disjoint.
2. Let $A_1 = \{1\}$, $A_2 = \{2\}$ and for $i \geq 1$, let $A_i = \{i\}$. Then the collection $A_1, A_2, \ldots$ is pairwise disjoint.

Definition (Power Set)

Let A be a set. The *power set* of A denoted $\mathcal{P}(A)$ is a set whose elements are the subsets of A. That is

$$\mathcal{P}(A) = \{S \quad | \quad S \subseteq A\}.$$

Example

Let $A = \{1, 2\}$. Then

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}.$$

Note

For any set A, we have $\emptyset, A \in \mathcal{P}(A)$ or $\{\emptyset, A\} \subseteq \mathcal{P}(A)$.

Definition (Set Difference and Complements)

1. We define the complement of B in A as
$$A - B = \{x \in A \quad | \quad x \notin B\}.$$
2. In may cases all sets under consideration will be subsets of some *universal set* U. In this case, we define the *complement* A' of a set $A \subseteq U$ as
$$A' = \{x \in U \quad | \quad x \notin A\}.$$

Example

Let

$$\begin{aligned} U &= \mathbb{Z}, \\ E &= \{x \in \mathbb{Z} \mid x \text{ is even}\}, \\ O &= \{x \in \mathbb{Z} \mid x \text{ is odd}\} \quad \text{and} \\ P &= \{x \in \mathbb{Z} \mid x > 0\}. \end{aligned}$$

Then

$$\begin{aligned} P - E &= \{1, 3, 5, 7, 9, \dots\} \\ E - P &= \{0, -2, -4, -6, \dots\} \\ E' &= O, \quad \text{and} \\ O' &= E. \end{aligned}$$

Definition (Partitions)

A separation of a set into nonempty pairwise disjoint sets is a partition.

Example

Suppose $A = \{1, 2, 3, 4, 5, 6\}$ and $X_1 = \{1, 4\}$, $X_2 = \{2, 3, 5\}$ and $X_3 = \{6\}$. Then $\mathcal{P} = \{X_1, X_2, X_3\}$ is a partition of A into 3 parts.

Facts

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
3. $(A \cap B)' = A' \cup B'$.
4. $(A \cup B)' = A' \cap B'$.