MTHSC 412 Section 4.2 – Cayley's Theorem

Kevin James

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Every group is isomorphic to a group of permutations. In particular, any group G is isomorphic to a subgroup of $\mathcal{S}(G)$, that is there is a monomorphism $\phi : G \to \mathcal{S}(G)$.

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$$\begin{aligned} f_0 &= e \\ f_1 &= \end{aligned}$$

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Then,

$$\begin{aligned} f_0 &= e \\ f_1 &= (0, 1, 2, 3, 4) \\ f_2 &= \end{aligned}$$

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So, $G' = \{e, (0,1,2,3,4), (0,2,4,1,3), (0,3,1,4,2), (0,4,3,2,1)\} \leq \mathcal{S}(\mathbb{Z}_5)$.