MTHSC 412 Section 4.5 – Normal Subgroups

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Definition

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Example

1. If G is abelian then $H \leq G$ if and only if $H \trianglelefteq G$.
2. A_3 is a normal subgroup of S_3.
3. $\{e, (1,2)\}$ is a non-normal subgroup of S_3.

Theorem

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Corollary

For any $H \leq G$, $H^2 = H$.

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Suppose that $H \leq G$*. Then* $H \trianglelefteq G$ *if and only if* $xHx^{-1} = H$ *for all* $x \in G$*.*

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Suppose that G is a group and that $\emptyset \neq A \subseteq G$. Then the *set generated by* A denoted $< A >$ is defined by

$$< A >= \{x \in G \mid x = a_1 a_2 \ldots a_n \text{ where either } a_i \in A \text{ or } a_i^{-1} \in A\}.$$

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Example

Let $G = S_3$ and let $A = \{(1,2),(1,3)\}$. Compute $< A >$.