MTHSC 412 Section 4.5 – Normal Subgroups

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Definition

Suppose that $H \leq G$. H is a *normal subgroup* or G if $xH = Hx$ for all $x \in G$. In this case, we will write $H \trianglelefteq G$.

Example

1. If G is abelian then $H \leq G$ if and only if $H \trianglelefteq G$.
2. A_3 is a normal subgroup of S_3.
3. $\{e, (1,2)\}$ is a non-normal subgroup of S_3.

Theorem

Suppose that $H \leq G$. Then $H = hH = Hh$ for all $h \in H$.

Corollary

For any $H \leq G$, $H^2 = H$.

Theorem

Suppose that $H \leq G$. Then $H \trianglelefteq G$ if and only if $xHx^{-1} = H$ for all $x \in G$.

Definition

Suppose that G is a group and that $\emptyset \neq A \subseteq G$. Then the *set generated by* A denoted $< A >$ is defined by

$$< A > = \{x \in G \mid x = a_1 a_2 \dots a_n \text{ where either } a_i \in A \text{ or } a_i^{-1} \in A\}.$$

Theorem

For any $\emptyset \neq A \subseteq G$, $< A > \leq G$.

Example

Let $G = S_3$ and let $A = \{(1,2),(1,3)\}$. Compute $< A >$.