

MTHSC 412 SECTION 4.2 – DIVISIBILITY IN $F[x]$

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DEFINITION

Suppose that F is a field and that $f, g \in F[x]$ with $f \neq 0$. We say that f divides g or is a factor of g and write $f|g$ if $g = fh$ for some $h \in F[x]$.

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NOTE

- 1 If $f|g$ then for any $0 \neq c \in F$, $(cf)|g$.
- 2 If $f|g$, then $\deg(f) \leq \deg(g)$.

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Let F be a field and suppose that $f, g \in F[x]$ are not both zero. Then the greatest common divisor of f and g is the monic polynomial d satisfying

- 1 $d|f$ and $d|g$.
- 2 If $c|f$ and $c|g$, then $\deg(c) \leq \deg(d)$.

THEOREM

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COROLLARY

Suppose that F is a field and that $f, g \in F[x]$ are not both zero. A monic polynomial $t \in F[x]$ is the greatest common divisor of f and g if and only if it satisfies the following conditions.

- 1 $d|f$ and $d|g$.
- 2 if $c|f$ and $c|g$ then $c|d$.

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NOTE

Since we have a division algorithm for $F[x]$, the Euclidean algorithm can be adapted to $F[x]$ and this gives an efficient method for computing the gcd and for writing the gcd as a combination of f and g .