MTHSC 412 Section 5.2 – Congruence Class Arithmetic

Kevin James

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THEOREM

Suppose that F is a field and that $p \in F[x]$ with deg(p) > 0. If [f] = [g] and [h] = [k] then,

[f + h] = [g + k] and [fh] = [gk].

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DEFINITION

Suppose that F is a field and that $p \in F[x]$ with $deg(p) \neq 0$. We define addition and multiplication on F[x]/(p) as follows.

$$[f] + [g] = [f + g]$$
 and $[f][g] = [fg]$.

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EXAMPLE

Give the addition and multiplication tables for $\mathbb{Z}_2[x]/(x^2 + x + 1)$.

Suppose that F is a field and that $p \in F[x]$ with $deg(p) \neq 0$. Then F[x]/(p) is a commutative ring with identity which contains a subring which is isomorphic to F.

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Remark

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Theorem

Suppose that F is a field and that $p \in F[x]$ with $\deg(p) \neq 0$. If $f \in F[x]$ and (f, p) = 1 then [f] is a unit in F[x]/(p).

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