MTHSC 412 Section 5.3 – The structure of $F[x]/(p)$ when p is irreducible

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Theorem

Suppose that F is a field and that $p \in F[x]$ with $\deg(p) \neq 0$. Then the following are equivalent.

1. *p is irreducible in $F[x]$.*
2. *$F[x]/(p)$ is a field.*
3. *$F[x]/(p)$ in an integral domain*

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Definition

Suppose that F is a field and that p is irreducible. We say that $F[x]/(p)$ is an extension field of F, since it is a field and it contains F.

Theorem

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Let F be a field and let $f \in F[x]$ with $\deg(f) > 0$. Then, there is an extension field K of F which contains a root of f.

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Note

$\mathbb{C} = \mathbb{R}[x]/(x^2 + 1)$.