MTHSC 412 SECTION 7.4 – ISOMORPHISMS AND HOMOMORPHISMS

Kevin James

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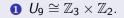
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Note

Isomorphism is an equivalence relation.



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- **6** $S_3 \not\cong \mathbb{Z}_6$.
- **6** Suppose that G is a group and $c \in G$. The map $f: G \to G$ given by $f(g) = c^{-1}gc$ is an <u>automorphism</u> of G called the inner automorphism of G induced by c.

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Suppose that G and H are groups with identity elements e_G and e_H respectively. Suppose also that $f:G\to H$ is a homomorphism. Then,

- **1** $f(e_G) = e_H$.
- 2 For all $a \in G$, $f(a^{-1}) = f(a)^{-1}$.
- $(f) \leq H.$
- **4** If f is injective, then $G \cong Im(f)$.

THEOREM (CAYLEY)

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COROLLARY

Every finite group G of order n is isomorphic to a subgroup of S_n .