MTHSC 412 Section 7.7 –Quotient Groups

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Notation

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Suppose that $N \trianglelefteq G$. Then,

1. *G/N is a group under the operation $(Na) * (Nb) = N(ab)$.*
2. *If G is finite then $|G/N| = [G : N] = \frac{|G|}{|N|}$.*
3. *If G is abelian then so is G/N.*

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   We saw last time that $N \trianglelefteq S_3$. Describe the group S_3/N.
2. Let $G = U_9$ and let $N =< 8 >$.
   Since G is abelian $N \trianglelefteq G$. Describe the group G/N.

Theorem

Suppose that $N \trianglelefteq G$. Then G/N is abelian if and only if $aba^{-1}b^{-1} \in N$ for all $a, b \in G$.

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If G is a group with $G/Z(G)$ cyclic, then G is abelian.