

# MTHSC 412 SECTION 7.7 – QUOTIENT GROUPS

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## NOTATION

If  $N \trianglelefteq G$ , then  $G/N$  denotes the set of right cosets of  $N$  in  $G$ . That is,  $G/N = \{Ng \mid g \in G\}$ .

## THEOREM

Suppose that  $N \trianglelefteq G$ . If  $Na = Nc$  and  $Nb = Nd$  in  $G/N$  then  $N(ab) = N(cd)$ .

## THEOREM

Suppose that  $N \trianglelefteq G$ . Then,

- 1  $G/N$  is a group under the operation  $(Na) * (Nb) = N(ab)$ .
- 2 If  $G$  is finite then  $|G/N| = [G : N] = \frac{|G|}{|N|}$ .
- 3 If  $G$  is abelian then so is  $G/N$ .

## DEFINITION

If  $N \trianglelefteq G$ , then the group  $G/N$  is called the quotient group or the factor group.

## EXAMPLE

- ① Let  $G = S_3$  and let  $N = \left\langle \left( \begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & 1 \end{array} \right) \right\rangle$ .

We saw last time that  $N \trianglelefteq S_3$ . Describe the group  $S_3/N$ .

- ② Let  $G = U_9$  and let  $N = \langle 8 \rangle$ .

Since  $G$  is abelian  $N \trianglelefteq G$ . Describe the group  $G/N$ .

## THEOREM

*Suppose that  $N \trianglelefteq G$ . Then  $G/N$  is abelian if and only if  $aba^{-1}b^{-1} \in N$  for all  $a, b \in G$ .*

## THEOREM

*If  $G$  is a group with  $G/Z(G)$  cyclic, then  $G$  is abelian.*