MTHSC 412 Section 7.8 –Quotient Groups and Homomorphisms

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Definition

Let $f : G \to H$ be a homomorphism of groups. We define the kernel of f as

$$\ker(f) = \{g \in G \mid f(g) = e_H\}.$$

Theorem

Let $f : G \to H$ be a homomorphism of groups. Then $\ker(f) \trianglelefteq G$.

Theorem

Let $f : G \to H$ be a homomorphism of groups. Then, f is injective if and only if $\ker(f) = \{e_G\}$.

Theorem

Let $N \trianglelefteq G$. Then the map $\pi : G \to G/N$ given by $\pi(g) = Ng$ is a surjective homomorphism with $\ker(\pi) = N$.

Theorem (First Isomorphism Theorem)

Let $f : G \to H$ be a surjective homomorphism of groups. Then $G/\ker(f) \cong H$.