Group Actions

Kevin James

Definition

Suppose that G is a group and A is a nonempty set. A <u>group action</u> of G on A is a function $\cdot : G \times A \to A$ satisfying the following axioms.

1. $g_1 (g_2 \cdot a) = (g_1 \cdot g_2) \cdot a$ for all $g_1, g_2 \in G$; $a \in A$.
2. $1_G \cdot a = a \; \forall a \in A$.

Proposition

Suppose that a group G acts on a set A. For all, $g \in G$, let $\sigma_g : A \to A$ be defined by $\sigma_g(a) = g \cdot a$. Then the map $g \mapsto \sigma_g$ is a homomorphism $G \to S_A$.

Example

1. Given any group G and any set A, we have the trivial action $g \cdot a = a$.
2. S_A acts on A.
3. D_{2n} acts on the regular n-gon.
4. $\mathbb{G}L_2(\mathbb{Z})$ acts on $\mathbb{C}$ by linear fractional transformations. Check this one.