Subgroups

Kevin James

Definition

Suppose that $(G, \cdot)$ is a group and the group axioms are satisfied when $\cdot$ is restricted to $H \subseteq G$. Then we say that H is a subgroup of G and write $H \leq G$.

Equivalently, we have

Proposition

If $(G, \cdot)$ is a group and $H \subseteq G$. Then $H \leq G$ if the following are true.

1. $H \neq \emptyset$.
2. $h_1 \cdot h_2 \in H$, $\forall h_1, h_2 \in H$.
3. $h^{-1} \in H$, $\forall h \in H$.

Example

1. For any group G, $\{e\}, G \leq G$.
2. $\{1, r, r^2, \ldots, r^{n-1}\} \leq D_{2n}$.
3. $\{\pm 1, \pm i\} \leq Q_8$

Proposition

Suppose that G is a group and that $\emptyset \neq H \subseteq G$. Then $H \leq G$ if and only if $h_1 h_2^{-1} \in H$, $\forall h_1, h_2 \in H$.