More on Cosets and Lagrange's Theorem

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Theorem (Lagrange's Theorem)

Suppose that G is a group and that $H \leq G$. Then $|H|$ divides $|G|$. In fact, if we denote the number of cosets of H in G by $[G : H]$ (also called the index of H in G), then we have $|G| = [G : H]|H|$.

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Note

Note that if G is a group and $H \leq G$ with $|H| = |G|/2$, then $H \trianglelefteq G$.

Definition

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Suppose that $H, K \leq G$. The set HK is a subgroup of G if and only if $HK = KH$.

Corollary

If $H, K \leq G$ and $H \leq N_G(K$, then $HK \leq G$. In particular, if $K \trianglelefteq G$, then $HK \leq G$ for all $H \leq G$.