Transpositions and the Alternating Group

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Definition

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Note

1. Recall that every permutation can be written as a product of disjoint cycles.
2. $(a_1, \ldots, a_m) = (a_1, a_m)(a_1, a_{m-1})(a_1, a_{m-2}) \ldots (a_1, a_3)(a_1, a_2)$.
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1. $\epsilon(\sigma) = \sigma(\Delta_n)/\Delta_n \quad \in \{\pm 1\}$.
2. $\epsilon(\sigma)$ is called the <u>sign</u> of σ.
3. σ is called an <u>even</u> permutation if $\epsilon(\sigma) = 1$ and an <u>odd</u> permutation if $\epsilon(\sigma) = -1$.

Proposition

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Fact

A_n is a non-Abelian simple group for all $n \geq 5$.