

GROUPS ACTING ON THEMSELVES BY LEFT MULTIPLICATION

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THEOREM

Suppose that G is a group and that $H \leq G$. Then G acts on the set A of left cosets of H by $g \cdot aH = (ga)H$. Let $\pi_H : G \rightarrow S_A$ be the associated permutation representation afforded by this action. Then,

- 1 G acts transitively on A ,
- 2 the stabilizer in G of the element $H \in A$ is the subgroup H and
- 3 the kernel of the action (which is $\ker(\pi_H)$) is given by $\bigcap_{x \in G} xHx^{-1}$, and $\ker(\pi_H)$ is the largest normal subgroup of G contained in H .

COROLLARY (CAYLEY'S THEOREM)

Every group is isomorphic to a subgroup of some symmetric group. If G is a group of order n , then G is isomorphic to a subgroup of S_n .

COROLLARY

If G is a finite group of order n and p is the smallest prime dividing n , then any subgroup of index p is normal.