Groups acting on themselves by left multiplication

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Theorem

Suppose that G is a group and that $H \leq G$. Then G acts on the set A of left cosets of H by $g \cdot aH = (ga)H$. Let $\pi_H : G \to S_A$ be the associated permutation representation afforded by this action. Then,

1. *G acts transitively on A,*
2. *the stabilizer in G of the element $H \in A$ is the subgroup H and*
3. *the kernel of the action (which is $\ker(\pi_H)$) is given by $\cap_{x \in G} xHx^{-1}$, and $\ker(\pi_H)$ is the largest normal subgroup of G contained in H.*

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Every group is isomorphic to a subgroup of some symmetric group. If G is a group of order n, then G is isomorphic to a subgroup of S_n.

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Corollary

If G is a finite group of order n and p is the smallest prime dividing n, then any subgroup of index p is normal.