PAlmetto **N**umber **T**heory **S**eries

SCHEDULE OF ACTIVITIES

Talks will take place in 100 Brackett Hall.
Coffee and refreshments will be available in 111 Brackett Hall during all breaks in addition to the times listed.

Saturday, December 5, 2015

8:30-9:00	Coffee and other refreshments
9:00-10:00	**Haruzo Hida** (UCLA), *André-Oort conjecture and non-vanishing of central L-values*
10:00 - 10:20	Break
10:20-10:40	**Abbey Bourdon** (University of Georgia), *Stratification of Torsion in Odd Degrees*
10:50 - 11:10	**Michael Mossinghoff** (Davidson College), *Double Wieferich pairs and circulant Hadamard matrices*
11:20 - 12:20	**Aaron Pollack** (Stanford University), *L-functions of Siegel modular forms*
12:20 - 2:30	Lunch
2:30 - 3:30	**Bianca Viray** (University of Washington), *Obstructions to the Hasse principle on Enriques surface*
3:30 - 3:50	Break

The organizers thank the National Science Foundation, the National Security Agency, and

the Mathematical Sciences Department at Clemson University for their support.

3:50 - 4:10 **Rodney Keaton** (University of Oklahoma), *An application of the Waldspurger model to a conjecture of Tonghai Yang*

4:20 - 4:40 **Richard Moy** (Northwestern University), *Non-CM Hilbert Modular forms of partial weight one*

4:50 - 5:10 **Ali Kemal Uncu** (University of Florida), *On partitions with fixed number of even-indexed and odd-indexed odd parts*

Sunday, December 6, 2015

8:30-9:00 Coffee and other refreshments

9:00-10:00 **Wei Zhang** (Columbia University), *Cycles on the moduli of Shtukas and Taylor coefficients of L-functions*

10:00 - 10:20 Break

10:20-10:40 **Jeremiah Bartz** (Francis Marion University), *On Areas of Fibonacci Polygons*

10:50 - 11:10 **Mckenzie West** (Emory University), *The Geometry of Some K3 Surfaces*

11:20 - 12:20 **Joe Kramer-Miller** (CUNY Graduate Center), *p-adic L-functions and the geometry of Hida families*

12:20 END OF CONFERENCE

Abstracts

JEREMIAH BARTZ, Francis Marion University, *On Areas of Fibonacci Polygons*

In this talk, we present a compact formula for computing the area of polygons whose vertices are comprised of consecutive Fibonacci numbers. In addition, we discuss related formulas for the area of triangles whose vertices involve certain types of sequences of Fibonacci and Lucas numbers.

ABBEY BOURDON, University of Georgia, *Stratification of Torsion in Odd Degrees*

Let E be an elliptic curve defined over a number field F. It is a classical theorem of Mordell and Weil that the collection of points of E with coordinates in F is a finitely generated abelian group. We seek to understand the subgroup of points with finite order, $E(F)$[tors], in the special case where E has complex multiplication (CM). In particular, given a positive integer d, we let $T(d)$ denote the set of isomorphism classes of abelian groups that appear as $E(F)$[tors] for some CM elliptic curve E defined over some number field F of degree d. If $T(d) = T(d)$, we say d is a d-Olson degree. For any odd d, we show the set of d-Olson degrees possesses a positive asymptotic density, and these densities sum to 1/2. This is joint work with Pete Clark and Paul Pollack.

HARUZO HIDA, UCLA, *André-Oort conjecture and non-vanishing of central L-values*

This is a joint work with A. Burungale. Assume the André-Oort conjecture for products of Hilbert modular varieties for a totally real field F (whose proof has been announced by Tsimerman, Yuan-Zhang and Andreatta-Goren-Howard-Pera), and pick a Hilbert modular Hecke eigen-cuspform with root number 1. We study non-vanishing of central critical values of $L(s, f \otimes \chi)$ twisted by conductor 1 Hecke characters of CM quadratic extensions K/F . As the discriminant of K tends to infinity, we prove that the number of characters χ with non-vanishing property also tends to infinity.

RODNEY KEATON, University of Oklahoma, *An application of the Waldspurger model to a conjecture of Tonghai Yang*

In a 2005 paper, Yang constructed families of Hilbert Eisenstein series, which when restricted to the diagonal are conjectured to span the underlying space of elliptic modular forms. One approach to these conjectures is to show the non-vanishing of an inner product of elliptic eigenforms with the restrictions of Eisenstein series. In this talk, I will present a new technique for computing such inner products.

JOE KRAMER-MILLER, CUNY, *p-adic L-functions and the geometry of Hida families*

A major theme in the theory of p-adic deformations of automorphic forms is how p-adic L-functions over eigenvarieties relate to the geometry of these eigenvarieties. In this talk we explain results in this vein for the ordinary part of the eigencurve (i.e. Hida families). We address how Taylor expansions of one variable p-adic L-functions varying over families can detect geometric phenomena: crossing components of a certain intersection multiplicity and ramification over the weight space. Our methods involve proving a converse to a result of Vatsal relating congruences between eigenforms to their algebraic special L-values and then p-adically interpolating congruences.

MICHAEL MOSSINGHOFF, Davidson College, *Double Wieferich pairs and circulant Hadamard matrices*

An $n \times n$ matrix, each of whose entries is ± 1, is *Hadamard* if its rows are mutually orthogonal; it is *circulant* if each row after the first is a cyclic shift to the right by one position of the prior row. A half-century-old question of Ryser asks if a circulant Hadamard matrix with order $n > 4$ exists. A number of necessary arithmetic conditions are known for a positive integer n to be the order of a circulant Hadamard matrix—the smallest integer $n > 4$ that passes all of these is $548,964,900$. We describe an algorithm for determining all integers up to a given bound that fulfill all of the known necessary conditions in this problem. We use this method to extend prior searches in this problem by a factor of 10^4, while reducing memory requirements by a factor of nearly 100. The principal improvement involves the incorporation of a special search for elusive double Wieferich prime pairs, which are pairs of primes $\{p, q\}$ for which $p^{q-1} \equiv 1 \bmod q^2$ and $q^{p-1} \equiv 1 \bmod p^2$. This is joint work with Brooke Logan, and represents a project from the 2014 Summer@ICERM REU program held at Brown University.

RICHARD MOY, Northwester University, *Non-CM Hilbert modular forms of partial weight one*

Hilbert modular forms are a generalization of classical modular forms to totally real extensions F of $\mathbb{Q}$. In 1997, Frazer Jarvis showed that each Hilbert cuspidal eigenform of partial weight one gives rise to a compatible system of 2-dimensional $\mathfrak{p}$-adic Galois representations of G_F where $\mathfrak{p}$ is a prime of F. However, at the time of his proof, the only known examples of such forms were those obtained through the automorphic-induction of a Grossencharacter. The construction of the Galois representations attached to these forms is classical and due to Hecke. This left open the possibility that Jarvis's construction was superfluous.

In this talk, I will discuss two results. First, I will construct the first example of a non-CM Hilbert cuspidal eigenform of partial weight one. This work is joint with Joel Specter. Second, I will show that for certain explicit fields and levels, all Hilbert cuspidal eigenforms of partial weight one are CM.

AARON POLLACK, Stanford, *L-functions of Siegel modular forms*

A Siegel modular form is a direct generalization of a modular form, whereby one replaces the group $GL(2) = GSp(2)$ with $GSp(2n)$ for general n. Associated to a Siegel modular form on $GSp(2n)$ is its Spin L-function, which for $n = 1$ is just the usual L-function of a modular form. While the basic analytic properties of the Spin L-function of Siegel modular forms on $GSp(2)$ and $GSp(4)$ are well-understand, this is not so for the Spin L-function of Siegel modular forms on $GSp(2n)$ with $n > 2$. I will discuss my work in the case of $GSp(6)$ to prove the finiteness of poles and functional equation of the Spin L-function of level one Siegel modular forms. The proof, which is via the Rankin-Selberg method, involves a construction Freudenthal used to explicitly realize the exceptional groups and the arithmetic invariant theory of orders in quaternion algebras.

ALI KEMAL UNCU, University of Florida, *On partitions with fixed number of even-indexed and odd-indexed odd parts*

In this talk we discuss partitions into distinct parts with fixed number of even/odd-indexed odd parts with restrictions on the size of parts. In particular we will see that the number of partitions into distinct parts with no odd odd-indexed part is equal to the number of partitions into distinct parts not congruent to 1 mod 4. We will later look at some implications of this study on partitions with fixed value of BG-rank. This talk is based on my recent work with Alexander Berkovich.

BIANCA VIRAY, University of Washington, *Obstructions to the Hasse principle on Enriques surface*

In 1970, Manin showed that the Brauer group can obstruct the existence of global rational points, even when there exist points everywhere locally. Later, Skorobogatov defined a refinement of this Brauer-Manin obstruction, called the tale-Brauer obstruction. We show that this refined obstruction is necessary to understand failures of the Hasse principle on Enriques surfaces, thereby completing the case of Kodaira dimension 0 surfaces. This is joint work with F. Balestrieri, J. Berg, M. Manes, and J. Park.

MCHENZIE WEST, Emory University, *The Geometry of Some K3 Surfaces*

Consider the following family of K3 surfaces

$$X_D : \; w^2 = x^6 + y^6 + z^6 + D(xyz)^2,$$

where $D \in \mathbb{Q}$. We wish to determine whether or not there is a Brauer-Manin obstruction to rational points on X_D by examining $\mathrm{Br}_1(X_D)/\mathrm{Br}_0(X_D) \simeq \mathrm{H}^1(\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}), \mathrm{Pic}(\overline{X}_D))$ where $\mathrm{Br}_1(X_D) := \ker(\mathrm{Br}(X_D) \to \mathrm{Br}(\overline{X}_D))$ and $\mathrm{Br}_0(X_D) := \mathrm{im}(\mathrm{Br}(\mathbb{Q}) \to \mathrm{Br}(X_D))$. We have shown that for a generic $D \in \mathbb{Q}$, $\mathrm{Pic}(\overline{X}_D)$ has rank 19 and $\mathrm{Br}_1(X_D)/\mathrm{Br}_0(X_D) \simeq (\mathbb{Z}/2\mathbb{Z})^3$. In this talk, I will discuss the strategy we used in proving these results and our ongoing research plans.

WEI ZHANG, Columbia University, *Cycles on the moduli of Shtukas and Taylor coefficients of L-functions*

This is a joint work with Zhiwei Yun. We prove a generalization of Gross-Zagier formula in the function field setting. Our formula relates self-intersection of certain cycles on the moduli of Shtukas for GL(2) to higher derivatives of L-functions.
