OPEN PROBLEMS – SERMON 2002

(1) *M. Filaseta*: Find for $\theta \in (0,1)$ an explicit $N = N(\theta)$ such that if $n \geq N(\theta)$ and $n(n+1) = 2^k 3^l 5^r * m$, then $m > n^\theta$.
(2) *M. Filaseta*: Consider $x^n - x - 1$. It has associated Galois group over $\mathbb{Q}$, the full symetric group S_n. Can we find $f(x)$ such that $x^n + f(x)$ has associated Galois group over $\mathbb{Q}$, the alternating group A_n for $n \geq 2$? (presumably the answer is no, perhaps one can solve by looking at a discriminant argument.)
(3) *Tom Brylowski*: This may be Martin Gardner's Impossible Problem? As given at SERMON 2002, pick $2 \leq a, b \leq N$ (for pencil/paper, let $N = 40$, for computer, let $N = 1000$.)

Mathematician Sum is given $a + b$, and tells Mathematician Product, who is given $a * b$, that "You don't know my sum." Product proceeds, after some thought, to tell Sum, "Yes, I do." After which, Sum tells Product, "Aha, I know your product." The question is: what are values of a and b? (This may be related to Goldbach's conjecture?) As a side note, Rhett Robinson looked up this problem on-line and found that the following site contains a good discussion of it:
http://www.mathematik.uni-bielefeld.de/%7Esillke/ PUZZLES/logic_sum_product
(4) *Theresa Vaughan*: Let S be a set of 3-sets in $P(n)$ such that no 5-set contains 3 or more of them. For $n \equiv 1, 3 \pmod 6$, the Steiner system does this and is not contained in a larger set. Among all possible configurations, does the Steiner system have the largest possible size? (This is known to be true for n up to 13, $n \equiv 1, 3 \pmod 6$.
(5) *Andrew Granville*: Along the curve $y = x^2$, the slope from point (b, b^2) to (a, a^2) is given by $a + b$. Consider this line over $\mathbb{F}_p$, and let T be the set of slopes of $\mathbb{F}_p^2$. ($T \subseteq \mathbb{F}_p \cup \{\infty\}$.) Find the minimal subset S of $\mathbb{F}_p^2$ such that $\forall t \in T, \exists$ points $q_1, q_2 \in S$ such that the slope from q_2 to q_1 is t.

Some ideas: $|S| > \sqrt{2p}$, maybe close? Also, find a subset of $\mathbb{F}_p$ such that the set of pairs gives all of $\mathbb{F}_p$.

Other thoughts: for $\mathbb{F}_p^3 \longrightarrow$ at least $\sqrt{2}p$ points for 3-dimensions? Also, what about the minimal $S \subseteq \mathbb{F}_p^k$ such that for all directions d, there exists a line in direction d contained in S?
(6) *Fred Portier*: Take a graph, and let $f(v)$ be the values of vertices, and let $w([a_i, b_i])$ be the weight of the edge connecting

a_i and b_i. The problem is to minimize $\sum_v f(v)$. such that $f(a_i) + f(b_i) \geq w([a_i, b_i])$.

Can this be considered from a linear programming perspective? Yes, though it is hard over the integers. Also, is there some graph theoretic algorithm that would be applicable to this problem? Is this problem NP hard?

(7) *Bud Brown*: Consier the following game, for example, with 6 points. Two players alternate between drawing a blue edge for player one or a red edge for player two to connect two points. Possible ways to win include being the first to complete a monochromatic triangle, or forcing the other to complete such a triangle. Is there a general winning strategy? Could this be related to the Ramsey theory? (e.g. $R(3,3) = 6$, and we're playing with 6 points, making triangles with 3 sides.) Since $R(4,4) = 18$, an alternative would be with 18 points, connecting 4 vertices monochromatically. Also, there could be 3-person games, using $R(3,3,3)$. In this style, are there cooperative strategies? Just look at everything. Another interesting idea–what about handicaps? Using $R(3,4)$ for example?

(8) *Andrew Granville*: For all $n \geq 2$, color 2 colors so that over a finite monochromatic set S, $\sum_{n \in S} \frac{1}{n} = 1$. Is there a winning strategy or a strategy to prevent winning?

(9) *Peter Fletcher*: **Def:** A pair (m, n) of numbers is E-symmetric provided $m - n = gcd(\phi(m), \phi(n))$. A number is E-symmetric iff it belongs to an E-symmetric pair; else it is E-asymmetric.

e.g. 10 is E-asymmetric; 23 is E-symmetric (it belongs to the pair $(23, 25)$.

Are there infinitely many:

(a) E-asymmetric numbers?
(b) E-asymmetric numbers that are prime?
(c) E-symmetric numbers that are prime? (e.g. 907 is the first prime that is E-asymmetric.)

This concludes the open questions rasied at SERMON 2002 in Clemson.