

In each case the polynomial given is the value of the  $p$ -rank of the orthogonal (dual) code of the design of point and  $r$ -dimensional subspaces over  $F_q$ , where  $q = p^t$  is a power of the prime  $p$ , in the projective space of dimension  $m$ . The degree is  $(q - 1)r$  and the coefficient of  $m^{(q-1)r}$  is  $\frac{t}{((q-1)r)!}$ .

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$$q = 4, r = 2$$

$$\frac{2}{6!}(m+1)(m+2)(m^4 + 18m^3 + 29m^2 + 72m + 180)$$


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$$q = 4, r = 3$$

$$\frac{2}{9!}(m+1)(m^8 + 44m^7 + 826m^6 + 1064m^5 + 9289m^4 + 25676m^3 + 85644m^2 + 149616m + 181440)$$


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$$q = 4, r = 4$$

$$\frac{2}{12!}(m+1)(m+2)(m^{10} + 75m^9 + 2490m^8 + 37590m^7 - 164247m^6 + 1245795m^5 + 167660m^4 + 8592060m^3 + 26605296m^2 + 43346880m + 119750400)$$


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$$q = 4, r = 5$$

$$\frac{2}{15!}(m+1)(m^{14} + 119m^{13} + 6461m^{12} + 181909m^{11} + 2735733m^{10} - 27390363m^9 + 226658003m^8 - 287580293m^7 + 2393897506m^6 + 5448887444m^5 + 35100765336m^4 + 92455219584m^3 + 296459386560m^2 + 548983008000m + 653837184000)$$


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$$q = 4, r = 6$$

$$\frac{2}{18!}(m+2)(m+1)(m^{16} + 168m^{15} + 13060m^{14} + 554736m^{13} + 13436374m^{12} + 165307968m^{11} - 5539922740m^{10} + 73291099728m^9 - 438573851551m^8 + 2073529633560m^7 - 4530978319000m^6 + 15864574614336m^5 + 12967596594576m^4 + 90381188306304m^3 + 383263652954880m^2 + 567413363865600m + 1600593426432000)$$


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$$q = 4, r = 7$$

$$\frac{2}{21!}(m+1)(m^{20} + 230m^{19} + 24795m^{18} + 1529310m^{17} + 57436506m^{16} + 1267975260m^{15} + 14063772070m^{14} - 985235601460m^{13} + 18063909964581m^{12} - 157011781481490m^{11} + 925909983165375m^{10} - 3095429863328010m^9 + 9832975608844816m^8 - 5173838215516720m^7 + 85392850884861360m^6 + 199017299982872160m^5 + 1383418617290868096m^4 + 3716165306079198720m^3 + 11374243844734310400m^2 + 21557619010013184000m + 25545471085854720000)$$


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$$q = 4, r = 8$$

$$\frac{2}{24!}(m+2)(m+1)(m^{22} + 297m^{21} + 41657m^{20} + 3426555m^{19} + 177442716m^{18} + 5730690042m^{17} + 105277470562m^{16} + 690891106950m^{15} - 198150319603159m^{14} + 5776269732740397m^{13} - 83669958539637483m^{12} + 791552042610904575m^{11} - 5142101134793948534m^{10} + 25179174440936347392m^9 - 88227989351191922848m^8 + 265253484517196015280m^7 - 435354887506633753824m^6 + 1269541561584515167872m^5 + 2026038087816252314112m^4 + 7509137834460241520640m^3 + 38265634599417140428800m^2 + 54575293730290679808000m + 155112100433309859840000)$$


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$$q = 8, r = 1$$

$$\frac{3}{7!}(m+1)(m^6 + 27m^5 + 295m^4 + 825m^3 + 1744m^2 + 2148m + 1680)$$


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$$q = 8, r = 2$$

$$\frac{3}{14!}(m+2)(m+1)(m^{12} + 102m^{11} + 4697m^{10} + 129030m^9 + 2353263m^8 + 29994426m^7 + 213181331m^6 + 528949410m^5 + 1498825636m^4 + 4977145272m^3 + 8664003072m^2 + 13144844160m + 14529715200)$$


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$$q = 8, r = 3$$

$$\frac{3}{21!}(m+1)(m^{20} + 230m^{19} + 24795m^{18} + 1664970m^{17} + 78056826m^{16} + 2714110860m^{15} + 72575557990m^{14} + 1519524165140m^{13} + 24975789135141m^{12} + 296234479265790m^{11} + 2094571157806335m^{10} + 3092495888499810m^9 + 37937916310602736m^8 + 124817683908495920m^7 + 552488014222165680m^6 + 1609891392776482080m^5 + 4701785318691175296m^4 + 10318877740334707200m^3 + 19034689212941875200m^2 + 23220102048933888000m + 170303140572364800000)$$


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$$q = 8, r = 4$$

$$\frac{3}{28!}(m + 2)(m + 1)(m^{26} + 403m^{25} + 77350m^{24} + 9410050m^{23} + 814656895m^{22} + 53417849485m^{21} + 2756902291000m^{20} + 114771333047800m^{19} + 3910733252961535m^{18} + 109821287136823405m^{17} + 2544108153603922750m^{16} + 48439412175480467050m^{15} + 725395182933252345265m^{14} + 7424483412511957542595m^{13} + 27976894224365706938500m^{12} - 285738046995173204989700m^{11} + 4445548431210952741527280m^{10} - 6305712569088753866405360m^9 + 49990467227260205635704000m^8 + 138099879084307731770836800m^7 + 416381021704786898067969024m^6 + 2360348597925294762783209472m^5 + 5382361320799704175230566400m^4 + 16107900065185474015561728000m^3 + 32257836013463841387479040000m^2 + 44918640562828634222100480000m + 50814724101952310083584000000)$$


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$$q = 8, r = 5$$

$$\frac{3}{35!}(m + 1)(m^{34} + 629m^{33} + 190366m^{32} + 36919784m^{31} + 5155689484m^{30} + 552258555596m^{29} + 47202023063184m^{28} + 3305380686582216m^{27} + 193102388529583830m^{26} + 9526343866318932510m^{25} + 399872088247290947340m^{24} + 14338277528377515197160m^{23} + 439652898084372371139180m^{22} + 11510127345821409673504620m^{21} + 256191595289095649061996480m^{20} + 4747178364749631317549078520m^{19} + 67916662890470289645213905265m^{18} + 576771000141695150459156182485m^{17} - 618431903629688824000835770810m^{16} - 69984784383992039088590422887440m^{15} + 1650331197380784798248943664708544m^{14} - 9024430915411930645861566856987424m^{13} + 47177485174962926569190600338704704m^{12} - 55671469062105142194592885399119104m^{11} + 477282807996628088660112992239854336m^{10} + 848459171109583338581847057212208384m^9 + 7931575115256799815803279512335668736m^8 + 24745256853739979929719344782290468864m^7 + 107793886427725050751439015150680719360m^6 + 331298312538356949382882819543159603200m^5 + 956384607525750616634968384670515200000m^4 + 2090255639315362083501722737748213760000m^3 + 3811006371637275666609752385807974400000m^2 + 4724613840761714190020443738472448000000m + 3444382655462048309888883779174400000000)$$


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$$q = 9, r = 1$$

$$\frac{2}{8!}(m+1)(m+2)(m^6 + 33m^5 + 445m^4 + 3135m^3 + 7114m^2 + 9432m + 10080)$$


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$$q = 9, r = 2$$

$$\frac{2}{16!}(m + 1)(m^{15} + 135m^{14} + 8365m^{13} + 315315m^{12} + 8078707m^{11} + 148873725m^{10} + 2036157695m^9 + 21021002145m^8 + 143137602608m^7 + 538812794520m^6 + 1275930459440m^5 + 3608050577040m^4 + 7656330893184m^3 + 13485570405120m^2 + 15114532608000m + 10461394944000)$$


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$$q = 9, r = 3$$

$$\frac{2}{16!}(m+3)(m+2)(m+1)(m^{21} + 294m^{20} + 40775m^{19} + 3547110m^{18} + 217077546m^{17} + 9935114364m^{16} + 352888691950m^{15} + 9963304105020m^{14} + 226720656078581m^{13} + 4175171164790094m^{12} + 61915308721874475m^{11} + 730273881191085630m^{10} + 6125341298104500496m^9 + 2536649010991259344m^8 + 26885942701524930800m^7 + 424257484869193513440m^6 + 1741099397685570389376m^5 + 3157857514019742395904m^4 + 12683891387466885888000m^3 + 25475132724320072908800m^2 + 34014467173874761728000m + 51704033477769953280000)$$


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$$q = 16, \ r = 1$$

$$\frac{4}{15!}(m+1)(m^{14} + 119m^{13} + 6461m^{12} + 211939m^{11} + 4687683m^{10} + 73870797m^9 + 854224943m^8 + 7093943857m^7 + 40012868896m^6 + 123817477784m^5 + 293768734896m^4 + 511468133904m^3 + 689704398720m^2 + 621631584000m + 326918592000)$$


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$$q = 25, \ r = 1$$

$$\frac{2}{24!}(m+4)(m+3)(m+2)(m+1)(m^{20} + 290m^{19} + 39615m^{18} + 3388650m^{17} + 203522946m^{16} + 9121022580m^{15} + 316404601630m^{14} + 8697685698500m^{13} + 192374726145381m^{12} + 3456380926339770m^{11} + 50707508702323395m^{10} + 608324168861056050m^9 + 5955504667302749896m^8 + 47306207576243088560m^7 + 301807600055278941360m^6 + 1522207900529046496800m^5 + 5386524779294396971776m^4 + 11761978590406197388800m^3 + 15849008498187131904000m^2 + 16828581707597721600000m + 12926008369442488320000)$$


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