1. Construct a realization of the code

$$\mathcal{C} = \{0000,\ 1000,\ 0100,\ 0010,\ 1100,\ 0110,\ 0101,\ 1101,\ 0111\}.$$

2. Let $\mathcal{C}$ be a code, and let σ and τ be subsets of $[n] = \{1, \dots, n\}$ such that $\sigma \cap \tau \neq \emptyset$.

   (a) Describe what the RF relationship

   $$\bigcap_{i \in \sigma} U_i \subseteq \bigcup_{j \in \tau} U_j$$

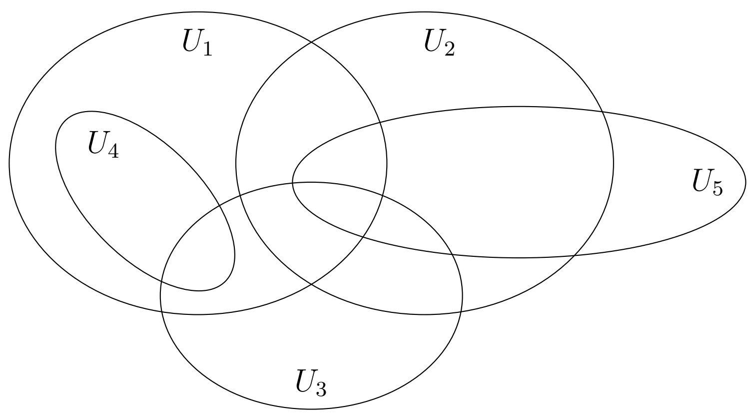
   captures, in a realization $\mathcal{U}$ of $\mathcal{C}$.

   (b) Construct an example of this for $n = 6$, where $\sigma = \{1, 2, 3, 4\}$ and $\tau = \{3, 4, 6\}$.

   (c) Express this condition algebraically, in terms of the neural ideal $J_{\mathcal{C}}$.

3. Show that the ideals $J = \langle x_1(1 - x_2)x_3, x_1x_2x_3\rangle$ and $K = \langle x_1x_3\rangle$ in $\mathbb{F}_2[x_1, x_2, x_3]$ are equal. Find a code $\mathcal{C}$ for which $J = J_{\mathcal{C}}$, sketch the place fields, and compute the canonical form.

4. Consider the place fields $\mathcal{U} = \{U_1, \dots, U_5\}$ shown below.



   (a) Write down the corresponding code $\mathcal{C}(\mathcal{U})$. Explain why it contains 15 binary strings, despite there being 16 connected regions.

   (b) Write down the vanishing ideal $I_{\mathcal{C}}$, and the neural ideal $J_{\mathcal{C}}$. Find a minimal generating set for each.

   (c) Construct the simplicial complex $\Delta(\mathcal{C})$.

   (d) Find another collection of place fields that has the same simplicial complex.

5. Explore the Neural Ideal Sage package, at `https://github.com/e6-1/NeuralIdeals`, or the Matlab package, available at `https://github.com/nebneuron/neural-ideal/`.