MTHSC 208, HW 5

(1) Section 2.5: # 6, 7, 9, 11-13.

(2) Section 2.9: # 16, 20, 22.

(3) Sketch a slope field for the equation \( y' = y(1 - y) \) and use this to sketch several solution curves.

(4) On the same diagram, sketch the solutions to \( y' = y(1 - y) \) and \( y' = 0.3y(1 - y) \) both satisfying the initial condition \( y(0) = 0.5 \).

   Explain how the difference in the equation explains the difference in these two curves.

(5) Let \( y' = ky(1 - \frac{y}{10}) \) and \( y'(0) = 2 \) and \( y(0) = 5 \).

   a. What is \( k \)? \( \text{Hint: No need to solve for } y(t) \text{ yet!} \)

   b. What is \( y(3) \)?

(6) The population of a certain planet is believed to be growing according to the logistic equation. The maximum population the planet can hold is \( 10^{10} \). In year zero the population is 50% of this maximum, and the rate of increase of the population is \( 10^9 \) per year.

   a. What is the logistic equation satisfied by the population, \( P(t) \)?

   b. How many years until the population reaches 90% of the maximum?

(7) A colony of bacteria is growing in a petri dish which has a maximum capacity of 100 mg. The mass of bacteria is increasing at a rate given by the logistic equation. Initially there is 2 mg of bacteria and the rate of increase is 1 mg per day.

   a. Write down the logistic equation satisfied by the mass, \( M(t) \).

   b. When will the mass of bacteria be 50 mg?

   c. What is the mass of bacteria 10 days after the mass was 2mg?