MTHSC 208, HW 14

(1) Find the general solution of $x^2y'' - xy' - 3y = 0$.

(2) Find the general solution of $x^2y'' - xy' + 5y = 0$.

(3) Find the general solution of $x^2y'' - 3xy' + 4y = 0$.

(4) Consider the following ODE: $y'' - 2xy' + 10y = 0$.

(a) Assume the solution is of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Plug $y(x)$ back into the ODE and find the recurrence relation for a_{n+2} in terms of a_n. Write down the general solution of the ODE.

(b) Explicitly write out the coefficients a_n for $n \leq 9$, in terms of a_0 and a_1. See the pattern? Write down the formula for a_n in terms of a_0 and a_1.

(c) Find a basis for the solution space of the ODE (functions $y_1(x)$ and $y_2(x)$ such that the general solution is $y(x) = C_1y_1(x) + C_2y_2(x)$).

(d) Find a non-zero polynomial solution for this ODE. *Hint: Make a good choice for a_0 and a_1.*

(e) Are there any other polynomial solutions, excluding scalar multiples of the one you found in (e)? Why or why not?