MTHSC 208, HW 15

(1) Use the ratio test to find the radius of convergence of the following power series:

a. $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^n,$ *b.* $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n+1}(x-\pi)^n,$ *c.* $\displaystyle\sum_{n=0}^{\infty}\frac{3}{n+1}(x-2)^n,$

d. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^n}(x-\pi)^n,$ *e.* $\displaystyle\sum_{n=0}^{\infty}(5x-10)^n,$ *f.* $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n!}(3x-6)^n.$

(2) Use the comparison test to find an estimate for the radius of convergence of each of the following power series:

a. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n},$ *b.* $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{2n},$

c. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n}(x-4)^{2n},$ *d.* $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^{2n}}(x-\pi)^{2n}.$

(3) Use the comparison test and the ratio test to find the radius of convergence of the power series

$$\sum_{n=0}^{\infty}(-1)^n\frac{1}{(n!)^2}\left(\frac{x}{2}\right)^{2n}.$$

(4) The differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+p(p+1)y=0,$$

where p is a constant, is known as *Legendre's equation.* We will solve it by first assuming that the solution is a power series centered at 0.

(a) Find the recursion formula for a_{n+2} in terms of a_n.
(b) Use the recursion formula to determine a_n in terms of a_0 and a_1, for $2 \le n \le 9$.
(c) Find a nonzero polynomial solution to this differential equation, in the case where $p = 3$.
(d) Find a basis for the space of solutions to the differential equation.

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+12y=0,$$

(5) The differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0,$$

where p is a constant, is known as Chebyshev's equation. It can be rewritten in the form

$$\frac{d^2y}{dx^2}-P(x)\frac{dy}{dx}+Q(x)y=0, \qquad \text{where} \qquad P(X)=-\frac{x}{1-x^2}, \quad Q(x)=\frac{p^2}{1-x^2}.$$

(a) If $P(x)$ and $Q(x)$ are represented as a power series about $x_0 = 0$, what is the radius of convergence of these power series?
(b) Assuming a power series centered at 0, find the recursion formula for a_{n+2} in terms of a_n.
(c) Use the recursion formula to determine a_n in terms of a_0 and a_1, for $2 \le n \le 9$.
(d) In the special case where $p = 3$, find a nonzero polynomial solution to this differential equation.