MTHSC 208, HW 18

If $f(t)$ is a 2π-periodic function, then $f(t)$ can be written *uniquely* as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \ , \tag{1}$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt \ , \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt \quad (n \geq 1) \ , \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt \quad (n \geq 1) \ .$$

The sum in (1) is the *Fourier series* of $f(t)$.

Exercises:

(1) The function

$$f(t) = \begin{cases} 0 & -\pi \leq t < \pi/2, \\ 1 & -\pi/2 \leq t < \pi/2, \\ 0 & \pi/2 \leq t \leq \pi, \end{cases}$$

can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

(2) The function

$$f(t) = |t|, \qquad \text{for } t \in [-\pi, \pi]$$

can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

(3) The function

$$f(t) = \begin{cases} 0 & -\pi \leq t < 0, \\ t & 0 \leq t \leq \pi, \end{cases}$$

can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

(4) Consider the 2π-periodic function defined by

$$f(t) = \begin{cases} t^2 & -\pi \leq t < \pi, \\ f(t - 2k\pi), & -\pi + 2k\pi \leq t < \pi + 2k\pi. \end{cases}$$

Sketch this function and compute its Fourier series.

(5) Find the Fourier series of the function $f(t) = 2 - 3\sin 4t + 5\cos 6t$, and sketch the graph of this function (use your calculator). *Hint: this problem is simple – don't do any integrals!*

(6) Sketch the graph of the function $f(t) = \sin^2 t$ and find its Fourier series. *Hint: Don't do any integrals! Instead, use a standard trig identity.*

(7) Which functions from the previous exercises had only cosine terms in their Fourier series expansion? Which functions only had sine terms? Which had both? Do you see a pattern? *Hint: compare the symmetries of the graphs of these functions to the symmetries of the graphs of sine waves and cosine waves.*