MTHSC 208 (Differential Equations)
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HW 1
Due Monday August 24th, 2009

(1) Let $P(t)$ be the net worth of an investment after t years, that is growing at a 5% rate. Suppose that after two years, the investment is worth \$200. Write down an initial value problem (differential equation & initial condition) for P, and sketch its solution.

(2) Let $T(t)$ be the temperature of a cup of water t minutes after being placed in a room where the ambient temperature is 72°.
   (a) Write down a differential equation that T satisfies.
   (b) Sketch the solution curve of the solution satisfying $T(0) = 100$.
   (c) Sketch the solution curve of the solution satisfying $T(0) = 40$.
   (d) Sketch the solution curve of the solution satisfying $T(0) = 72$.

(3) Repeat the previous exercise, except let $T(t)$ be the temperature of a sheet of metal (which cools down and heats up *much* quicker than water). Qualitatively, what is the difference between the solution curves in these two problems? Which value of k is bigger and why?

(4) Sketch the slope field of the ODE $y' = t - 2y$ using the isocline method for $c = 0, \pm 1, \pm 2, \pm 3$. Sketch the particular solutions that satisfy $y(0) = 1$ and $y(2) = 2$.

(5) Sketch the slope field of the ODE $y\,y' = -t$ using the isocline method for $c = 0, \pm\frac{1}{2}, \pm 1, \pm 2$, and sketch the particular solution that satisfies $y(0) = 1$.

(6) Explain why two solution curves in the slope field of an ODE can never intersect.

(7) Sketch the steady-state (constant) solutions of $y' = 6 + y - y^2$ in the ty-plane. These solutions divide the plane into regions. Sketch at least one solution curve in each of these region.

(8) Consider the differential equation $y' = y(4 - y)$.
   (a) Show that $y(t) = 4/(1 + Ce^{-4t})$ is a solution for any value of C. This family of solutions is called a *general solution* to the differential equation.
   (b) Sketch the solutions for $C = 1, 2, \ldots, 5$. (Hint: This ODE is autonomous).
   (c) What are the steady-state (constant) solutions?
   (d) The general solution may fail to produce all solutions of a differential equation. Find a solution that is not given by any value of C. (Hint: Look at part (c)).
   (e) Describe a physical situation that this differential equation could model, and justify your reasons. (Hint: Consider population growth).