(1) Let $P(t)$ be the net worth of an investment after $t$ years, that is growing at a 5% rate. Suppose that after two years, the investment is worth $200. Write down an initial value problem (differential equation & initial condition) for $P$, and sketch its solution.

(2) Let $T(t)$ be the temperature of a cup of water $t$ minutes after being placed in a room where the ambient temperature is 72°.

(a) Write down a differential equation that $T$ satisfies.

(b) Sketch the solution curve of the solution satisfying $T(0) = 100$.

(c) Sketch the solution curve of the solution satisfying $T(0) = 40$.

(d) Sketch the solution curve of the solution satisfying $T(0) = 72$.

(3) Repeat the previous exercise, except let $T(t)$ be the temperature of a sheet of metal (which cools down and heats up much quicker than water). Qualitatively, what is the difference between the solution curves in these two problems? Which value of $k$ is bigger and why?

(4) Sketch the slope field of the ODE $y' = t - 2y$ using the isocline method for $c = 0, \pm 1, \pm 2, \pm 3$.

Sketch the particular solutions that satisfy $y(0) = 1$ and $y(2) = 2$.

(5) Sketch the slope field of the ODE $yy' = -t$ using the isocline method for $c = 0, \pm \frac{1}{2}, \pm 1, \pm 2$, and sketch the particular solution that satisfies $y(0) = 1$.

(6) Explain why two solution curves in the slope field of an ODE can never intersect.

(7) Sketch the steady-state (constant) solutions of $y' = 6 + y - y^2$ in the $ty$-plane. These solutions divide the plane into regions. Sketch at least one solution curve in each of these region.

(8) Consider the differential equation $y' = y(4 - y)$.

(a) Show that $y(t) = 4/(1 + Ce^{-4t})$ is a solution for any value of $C$. This family of solutions is called a general solution to the differential equation.

(b) Sketch the solutions for $C = 1, 2, \ldots, 5$. (Hint: This ODE is autonomous).

(c) What are the steady-state (constant) solutions?

(d) The general solution may fail to produce all solutions of a differential equation. Find a solution that is not given by any value of $C$. (Hint: Look at part (c)).

(e) Describe a physical situation that this differential equation could model, and justify your reasons. (Hint: Consider population growth).