MTHSC 208 (Differential Equations)
Dr. Matthew Macauley
HW 2
Due Monday August 31st, 2009

(1) Consider the initial value problem $y' = t + y$, $y(0) = 1$.
   (a) When computing a solution by hand using Euler's method, it is beneficial to arrange your work in a table, as shown below where the first step is computed. Continue with Euler's method using step-size $h = 0.1$ and complete all missing entries of the table.

k	t_k	y_k	$f(t_k, y_k) = t_k + y_k$	h	$f(t_k, y_k) \cdot h$
0	0.0	1.0	1.0	0.1	0.1
1	0.1	1.1			
2	0.2				
3	0.3				
4	0.4				
5	0.5				

   (b) Compute the actual value of $y(0.5)$ by solving the initial value problem $y' = t + y$, $y(0) = 1$, and plugging in $t = 0.5$.

(2) Consider the initial value problem $y' = (1 + t)y$, $y(0) = -1$.
   (a) Use Euler's method to approximate $y(1)$, for step-size $h = 0.2$, and then for $h = 0.1$. Arrange your results in the tabular form as in the previous exercise.
   (b) Compute the actual value of $y(1)$ by solving the initial value problem $y' = (1 + t)y$, $y(0) = -1$ and plugging in $t = 1$.

(3) Find the general solution of the following differential equations.
   (a) $y' = ty$
   (b) $ty' = 2y$
   (c) $y' = e^{t-y}$

(4) Find the particular solution of the following initial value problems.
   (a) $y' = y/t, \quad y(1) = -2$
   (b) $y' = -2t(1 + y^2)/y, \quad y(0) = 1$
   (c) $y' = (\sin t)/y, \quad y(\pi/2) = 1$

(5) Suppose that \$1200 is invested at a rate of 5%, compounded continuously.
   (a) Assuming no additional withdrawals or deposits, how much will be in the account after 10 years?
   (b) How long will it take the balance to reach \$5000?

(6) Tritium is an isotope of hydrogen that is sometimes used as a biochemical tracer. Suppose that 100 mg of tritium decays to 80 mg in 4 hours. Determine its half-life.

(7) Suppose a cold beer at 40°F is placed into a warm room at 70°F. Suppose 10 minutes later, the temperature of the beer if 48°F. Use Newton's law of cooling to find the temperature 25 minutes after the beer was placed into the room.

(8) A murder victim is discovered at midnight at the temperature of the body is recorded at 31°C. One hour later, the temperature of the body is 29°C. Assume that the surrounding air temperature remains constant at 21°C. Use Newton's law of cooling (the differential equation $T' = k(A-T)$) to calculate the victim's time of death (when his body temperature was 37°C).