MTHSC 208 (Differential Equations) Dr. Matthew Macauley HW 7 Due Friday September 18th, 2009

For full credit, be sure to show your work on all of these problems!

- (1) Find the general solution to the following 2^{nd} order linear homogeneous ODEs.
 - (a) y'' + 2y = 0(b) y'' + 4y' + 5y'
 - (b) y'' + 4y' + 5y = 0
- (2) Find the particular solution to the following 2^{nd} order linear ODEs.
 - (a) y'' + 25y = 3, y(0) = 1, y'(0) = -1
 - (b) y'' 2y' + 17y = 0, y(0) = -2, y'(0) = 3
- (3) As we've seen, to solve ODE of the form

$$y'' + py' + qy = 0$$
, p and q constants

we assume that the solution has the form e^{rt} , and then we plug this back into the ODE to get the *characteristic equation*: $r^2 + pr + q = 0$. Given that this equation has a double root $r = r_1$ (i.e., the roots are $r_1 = r_2$), show by direct substitution that $y = te^{r_1 t}$ is a solution of the ODE, and then write down the general solution.

(4) Suppose that z(t) = x(t) + iy(t) is a solution of

$$z'' + pz' + qz = Ae^{i\omega t}$$

Substitute z(t) into this equation above. Then compare (equate) the real and imaginary parts of each side to prove two facts:

$$x'' + px' + qx = A\cos\omega t$$
$$y'' + py' + qy = A\sin\omega t.$$

Write a sentence or two summarizing the significance of this result.

- (5) Solve the following initial value problems using the method of undetermined coefficients.
 - (a) $y'' + 3y' + 2y = -3e^{-4t}$, y(0) = 1, y'(0) = 0
 - (b) $y'' + 2y' + 2y = 2\cos 2t$, y(0) = -2, y'(0) = 0
 - (c) y'' + 4y' + 4y = 4 t, y(0) = -1, y'(0) = 0
 - (d) $y'' 2y' + y = t^3$, y(0) = 1, y'(0) = 0