MTHSC 208 (Differential Equations)
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HW 10
Due Monday October 5th, 2009

For full credit, be sure to *show your work* on all of these problems!

(1) For each system below, find all solutions, and sketch the graph of the lines in each system on the same axis. Are the resulting lines intersecting, parallel, or coincident?
   (a) $x_1 + 3x_2 = 0, \quad 2x_1 - x_2 = 0$
   (b) $-x_1 + 2x_2 = 4, \quad 2x_1 - 4x_2 = -6$
   (c) $2x_1 - 3x_2 = 4, \quad x_1 + 2x_2 = -5$
   (d) $3x_1 - 2x_2 = 0, \quad -6x_1 + 4x_2 = 0$
   (e) $2x_1 - 3x_2 = 6, \quad -4x_1 + 6x_2 = -12$

(2) For each problem, find the eigenvalues and eigenvectors of the given matrix.
   (a) $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$ (c) $\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$
   (d) $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}$ (e) $\mathbf{A} = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$ (f) $\mathbf{A} = \begin{pmatrix} 5/4 & 3/4 \\ -3/4 & -1/4 \end{pmatrix}$

(3) For each problem below, find the eigenvalues of the given matrix, and then describe how the nature of the eigenvalue depends on the parameter α.
   (a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & \alpha \end{pmatrix}$ (b) $\mathbf{x}' = \begin{pmatrix} 1 & -\alpha \\ 2\alpha & 3 \end{pmatrix}$

(4) Show that $\lambda = 0$ is an eigenvalue of the matrix $\mathbf{A}$ if and only if $\det(\mathbf{A}) = 0$.