MTHSC 208 (Differential Equations)
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HW 15
Due Friday November 6th, 2009

For full credit, be sure to *show your work* on all of these problems!

(1) Find the Laplace transform of the given function.
   (a) $3H(t-2)$
   (b) $(t-2)H(t-2)$
   (c) $e^{2(t-1)}H(t-1)$
   (d) $H(t-\pi/4)\sin 3(t-\pi/4)$
   (e) $t^2H(t-1)$
   (f) $e^{-t}H(t-2)$
   (g) $\sin(2t)H(t-\pi/6)$

(2) In this exercise, you will examine the effect of shifts in the time domain on the Laplace transform (graphically).
   (a) Sketch the graph of $f(t) = \sin t$ in the time domain. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}(s)$. Sketch the graph of F in the s-domain on the interval $[0, 2]$.
   (b) Sketch the graph of $g(t) = H(t-1)\sin(t-1)$ in the time domain. Find the Laplace transform $G(s) = \mathcal{L}\{g(t)\}(s)$. Sketch the graph of G in the s-domain on the interval $[0, 2]$ on the same axes used to sketch the graph of F.
   (c) Repeat the directions in part (b) for $g(t) = H(t-2)\sin(t-2)$. Explain why engineers like to say that "a shift in the time domain leads to an attenuation (scaling) in the s-domain."

(3) Use the Heaviside function to redefine each piecewise function.
   (a) $f(t) = \begin{cases} 5, & 2 \leq t < 4; \\ 0, & \text{otherwise} \end{cases}$
   (b) $f(t) = \begin{cases} 0, & t < 0; \\ t, & 0 \leq t < 3 \\ 4 & t \geq 3 \end{cases}$
   (c) $f(t) = \begin{cases} 0, & t < 0; \\ t^2, & 0 \leq t < 2 \\ 4 & t \geq 2 \end{cases}$

(4) Find the inverse Laplace transform of each function. Create a piecewise definition for your solution that doesn't use the Heavyside function.
   (a) $F(s) = \dfrac{e^{-2s}}{s+3}$
   (b) $F(s) = \dfrac{1-e^{-s}}{s^2}$
   (c) $F(s) = \dfrac{e^{-s}}{s^2+4}$