MTHSC 208 (Differential Equations)
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HW 17
Due Friday November 13th, 2009

(1) The function

$$f(x) = \begin{cases} 0 & -\pi \le x < -\pi/2, \\ 1 & -\pi/2 \le x < \pi/2, \\ 0 & \pi/2 \le x \le \pi, \end{cases}$$

can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

(2) The function

$$f(t) = |x|, \qquad \text{for } x \in [-\pi, \pi]$$

can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

(3) The function

$$f(x) = \begin{cases} 0 & -\pi \le x < 0, \\ x & 0 \le x \le \pi, \end{cases}$$

can be extended to be periodic of period 2π. Sketch the graph of the resulting function, and compute its Fourier series.

(4) Consider the 2π-periodic function defined by

$$f(x) = \begin{cases} x^2 & -\pi \le x < \pi, \\ f(x - 2k\pi), & -\pi + 2k\pi \le x < \pi + 2k\pi. \end{cases}$$

Sketch this function and compute its Fourier series.

(5) Find the Fourier series of the following functions *without* computing any integrals.
  (a) $f(x) = 2 - 3\sin 4x + 5\cos 6x$,
  (b) $f(x) = \sin^2 x$ [*Hint*: Use a standard trig identity.]

(6) Determine which of the following functions are even, which are odd, and which are neither even nor odd:
  (a) $f(t) = x^3 + 3x$.
  (b) $f(t) = x^2 + |x|$.
  (c) $f(t) = e^x$.
  (d) $f(t) = \frac{1}{2}(e^x + e^{-x})$.
  (e) $f(t) = \frac{1}{2}(e^x - e^{-x})$.