MTHSC 208 (Differential Equations)
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HW 18
Due Monday November 16th, 2009

(1) Suppose that f is a function defined on $\mathbb{R}$ (not necessarily periodic). Show that there is an odd function f_{odd} and an even function f_{even} such that $f(x) = f_{\text{odd}} + f_{\text{even}}$. *Hint*: As a guiding example, suppose $f(x) = e^x$, and consider $\cos x = \frac{1}{2}(e^x + e^{-x})$ and $i \sin x = \frac{1}{2}(e^x - e^{-x})$.

(2) Express the y-intercept of $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$ in terms of the a_n's and b_n's. (*Hint*: It's *not* a_0 or $a_0/2$!)

(3) Consider the 2π-periodic function $f(t) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$. Write the Fourier series for the following functions:
  (a) The reflection of $f(x)$ across the y-axis;
  (b) The reflection of $f(x)$ across the x-axis;
  (c) The reflection of $f(x)$ across the origin.

(4) (a) The Fourier series of an odd function consists only of sine-terms. What additional symmetry conditions on f will imply that the sine coefficients with even indices will be zero (i.e., each $b_{2n} = 0$)? Give an example of a non-zero function satisfying this additional condition.
  (b) What symmetry conditions on f will imply that the sine coefficients with odd indices will be zero (i.e., each $b_{2n+1} = 0$)? Give an example of a non-zero function satisfying this additional condition.
  (c) Sketch the graph of a non-zero even function, such that $a_{2n} = 0$ for all n.
  (d) Sketch the graph of a non-zero even function, such that $a_{2n+1} = 0$ for all n.

(5) Consider the function defined on the interval $[0, \pi]$:

$$f(x) = \begin{cases} x & \text{for } 0 \le x < \pi/2, \\ \pi - x, & \text{for } \pi/2 \le x \le \pi. \end{cases}$$

  (a) Sketch the even extension of this function and find its Fourier cosine series.
  (b) Sketch the odd extension of this function and find its Fourier sine series.

(6) Consider the function defined on the interval $[0, \pi]$:

$$f(x) = x(\pi - x).$$

  (a) Sketch the even extension of this function and find its Fourier cosine series.
  (b) Sketch the odd extension of this function and find its Fourier sine series.